Work input, W

$$= C_p(T_2' - T_i) - C_p(T_i - T_2)$$

$$= C_p(T_2' + T_2 - 2T_i) \quad (1)$$

Now, the entropy change of body A

$$\Delta S_A = \int_{T_i}^{T_2} C_p \frac{dT}{T} = C_p \ln \frac{T_2}{T_i} \text{ (negative)}$$

The entropy change of body B

$$\Delta S_B = \int_{T_i}^{T_2'} C_p \frac{dT}{T} = C_p \ln \frac{T_2'}{T_i} \text{ (positive)}$$

Entropy change (cycle) of refrigerant = 0

∴ Entropy change of the universe

$$(\Delta S)_{univ} = \Delta S_A + \Delta S_B$$

$$= C_p \ln \frac{T_2}{T_i} + C_p \ln \frac{T_2'}{T_i}$$

By the entropy principle

$$(\Delta S)_{univ} \geq 0$$

$$\left(C_p \ln \frac{T_2}{T_i} + C_p \ln \frac{T_2'}{T_i}\right) \geq 0$$

$$C_p \ln \frac{T_2 T_2'}{\cdot T_i^2} \geq 0 \quad (2)$$

In Eq. (7.4.1) with C_p, T_2, and T_i being given, W will be a minimum when T_2' is a minimum. From Eq. (2), the minimum value of T_2' would correspond to

$$C_p \ln \frac{T_2 T_2'}{T_i^2} = 0 = \ln 1$$

$$\therefore \quad T_2' = \frac{T_i^2}{T_2}$$

From Eq. (1)

$$W_{(min)} = C_p \left(\frac{T_i^2}{T_2} + T_2 - 2T_i\right) \quad \text{Proved.}$$

Example 7.5

Three identical finite bodies of constant heat capacity are at temperatures 300, 300, *and* 100 K. *If no work or heat is supplied from outside, what is the highest temperature to which any one of the bodies can be raised by the operation of heat engines or refrigerators?*

Solution Let the three identical bodies A, B, and C having the same heat capacity C be respectively at 300 K, 100 K and 300 K initially, and let us operate a heat engine and a refrigerator, as shown in Fig. Ex. 7.5. Let T_f be the final temperature of bodies A and B, and T_f' be the final temperature of body C. Now

$$(\Delta S)_A = C \ln \frac{T_f}{300}$$

$$(\Delta S)_B = C \ln \frac{T_f}{300}$$

$$(\Delta S)_C = C \ln \frac{T_f'}{300}$$

$$(\Delta S)_{H.E.} = 0$$

$$(\Delta S)_{ref} = 0$$

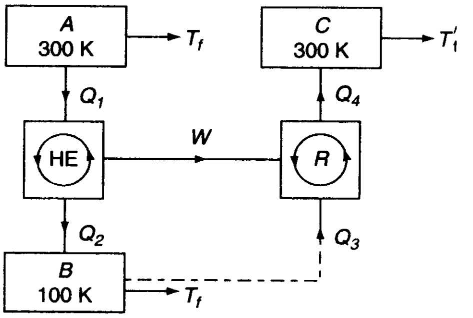


Fig. Ex. 7.5

where C is the heat capacity of each of the three bodies.

Since $$(\Delta S)_{univ} \geq 0$$

$$\left(C \ln \frac{T_f}{300} + C \ln \frac{T_f}{100} + C \ln \frac{T_f'}{300} \right) \geq 0$$

$$C \ln \frac{T_f^2 T_f'}{9{,}000{,}000} \geq 0$$

For minimum value of T_f, $C \ln \dfrac{T_f^2 T_f'}{9 \times 10^6} = 0 = \ln 1$

$$\therefore \quad T_f^2 T_f' = 9{,}000{,}000 \tag{1}$$

Now

$$Q_1 = C(300 - T_f)$$

$$Q_2 = C(T_f - 100)$$

$$Q_4 = C(T_f' - 300)$$

Again

$$\begin{aligned} Q_1 &= \text{Heat removed from body } A \\ &= \text{Heat discharged to bodies } B \text{ and } C \\ &= Q_2 + Q_4 \end{aligned}$$

$$\therefore \quad C(300 - T_f) = C(T_f - 100) + C(T_f' - 300)$$

$$\therefore \quad T_f' = 700 - 2T_f \tag{2}$$

T_f' will be the highest value when T_f is the minimum.

From Eqs (1) and (2)

$$T_f^2 (700 - 2T_f) = 9{,}000{,}000$$

$$\therefore \quad 2T_f^3 - 700\, T_f^2 + 9{,}000{,}000 = 0$$

or $$T_f = 150 \text{ K}$$

From Eq. (2)

$$T'_f = (700 - 2 \times 150)\text{ K}$$
$$= 400\text{ K} \qquad \textit{Ans.}$$

Example 7.6

A system has a heat capacity at constant volume

$$C_V = AT^2$$

where $A = 0.042$ *J/K*^{3}.

The system is originally at 200 *K, and a thermal reservoir at* 100 *K is available. What is the maximum amount of work that can be recovered as the system is cooled down to the temperature of the reservoir?*

Solution Heat removed from the system (Fig. Ex.7.6)

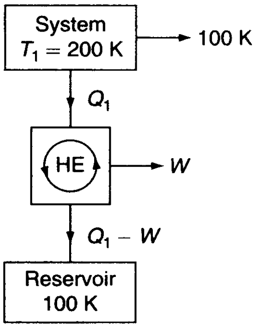


Fig. Ex. 7.6

$$Q_1 = \int_{T_1}^{T_2} C_V\,\mathrm{d}T = \int_{T_1 = 200\text{ K}}^{T_2 = 100\text{ K}} 0.042\,T^2\,\mathrm{d}T$$

$$= 0.042\left[\frac{T^3}{3}\right]_{200\text{ K}}^{100\text{ K}}$$

$$= \frac{0.042}{3}\text{ J/K}^3\,(100^3 - 200^3)\text{K}^3 = -98 \times 10^3\text{ J}$$

$$(\Delta S)_{\text{system}} = \int_{200\text{K}}^{100\text{K}} C_V\frac{\mathrm{d}T}{T} = \int_{200\text{K}}^{100\text{K}} 0.042\,T^2\frac{\mathrm{d}T}{T}$$

$$= \frac{0.042}{2}\text{ J/K}^3\,[100^2 - 200^2]\text{K}^2 = -630\text{ J/K}$$

$$(\Delta S)_{\text{res}} = \frac{Q_1 - W}{T_{\text{res}}} = \frac{98 \times 10^3 - W}{100}\text{ J/K}$$

$$(\Delta S)_{\text{working fluid in H.E.}} = 0$$

$$\therefore \quad (\Delta S)_{\text{univ}} = (\Delta S)_{\text{system}} + (\Delta S)_{\text{res}} = -630 + \frac{98 \times 10^3 - W}{100}$$

Since $(\Delta S)_{\text{univ}} \geq 0$

$$\therefore \quad -630 + \frac{98 \times 10^3 - W}{100} \geq 0$$

$$980 - \frac{W}{100} - 630 \geq 0$$

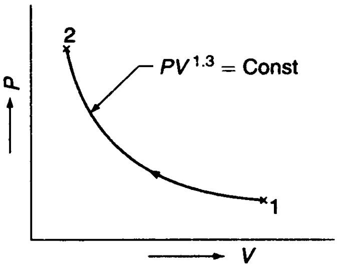
$$\frac{W}{100} \leq 350$$

$$W_{(\text{max})} = 35{,}000\text{ J} = 35\text{ kJ} \qquad \textit{Ans.}$$

Example 7.7

A fluid undergoes a reversible adiabatic compression from 0.5 MPa, 0.2 m³ to 0.05 m³ according to the law, $pv^{1.3}$ = constant. Determine the change in enthalpy, internal energy and entropy, and the heat transfer and work transfer during the process.

Solution



[illegible]

$$TdS = dH - Vdp$$

For the reversible adiabatic process (Fig. Ex. 7.7)

$$dH = Vdp$$

$$p_1 = 0.5 \text{ MPa}, \quad V_1 = 0.2 \text{ m}^3$$

$$V_2 = 0.05 \text{ m}^3, \quad p_1 V_1^n = p_2 V_2^n$$

$$\therefore \quad p_2 = p_1 \left(\frac{V_1}{V_2}\right)^n$$

$$= 0.5 \times \left(\frac{0.20}{0.05}\right)^{1.3} \text{ MPa}$$

$$= 0.5 \times \left(\frac{0.20}{0.05}\right)^{1.3} \text{ MPa} = 0.5 \times 6.061 \text{ MPa} = 3.0305 \text{ MPa}$$

$$p_1 V_1^n = pV^n$$

$$\therefore \quad V = \left(\frac{p_1 V_1^n}{p}\right)^{1/n}$$

$$\int_{H_1}^{H_2} dH = \int_{p_1}^{p_2} Vdp$$

$$H_2 - H_1 = \int_{p_1}^{p_2} \left[\left(\frac{p_1 V_1^n}{p}\right)\right]^{1/n} dp$$

$$= (p_1 V_1^n)^{1/n} \left(\frac{p_1^{1-1/n} - p_1^{1-1/n}}{1 - 1/n}\right)$$

$$= \frac{n(p_2 V_2 - p_1 V_1)}{n - 1}$$

$$= \frac{1.3(3030.5 \times 0.05 - 500 \times 0.2)}{1.3 - 1} = 223.3 \text{ kJ}$$

$$H_2 - H_1 = (U_2 + p_2 V_2) - (U_1 + p_1 V_1)$$

$$= (U_2 - U_1) + (p_2 V_2 - p_1 V_1)$$

$$\therefore \quad U_2 - U_1 = (H_2 - H_1) - (p_2 V_2 - p_1 V_1)$$

$$= 223.3 - 51.53 = 171.77 \text{ kJ}$$ *Ans.*

$$S_2 - S_1 = 0$$ *Ans.*

$$Q_{1-2} = 0$$ *Ans.*

$$Q_{1-2} = U_2 - U_1 + W_{1-2}$$

$$\therefore \quad W_{1-2} = U_1 - U_2 = -171.77 \text{ kJ}$$ *Ans.*

Example 7.8

Air is flowing steadily in an insulated duct. The pressure and temperature measurements of the air at two stations A and B are given below. Establish the direction of the flow of air in the duct. Assume that for air, specific heat c_p is constant at 1.005 kJ/kg K, $h = c_p T$, and $\frac{v}{T} = \frac{0.287}{p}$, where p, v, and T are pressure (in kPa), volume (in m^3/kg) and temperature (in K) respectively.

	Station A	*Station B*
Pressure	130 kPa	100 kPa
Temperature	50°C	13°C

Solution From property relation

$$T ds = dh - v dp$$

$$ds = \frac{dh}{T} - v\frac{dp}{T}$$

For two states at A and B the entropy change of the system

$$\int_{s_A}^{s_B} ds = \int_{T_A}^{T_B} \frac{c_p dT}{T} - \int_{p_A}^{p_B} 0.287 \frac{dp}{p}$$

$$\therefore \quad s_B - s_A = 1.005 \ln \frac{T_B}{T_A} - 0.287 \ln \frac{p_B}{p_A}$$

$$= 1.005 \ln \frac{273 + 13}{273 + 50} - 0.287 \ln \frac{100}{130}$$

$$= -0.1223 + 0.0753 = -0.047 \text{ kJ/kg K}$$

$$(\Delta s)_{\text{system}} = -0.047 \text{ kJ/kg K}$$

Since the duct is insulated $(\Delta s)_{\text{surr}} = 0$

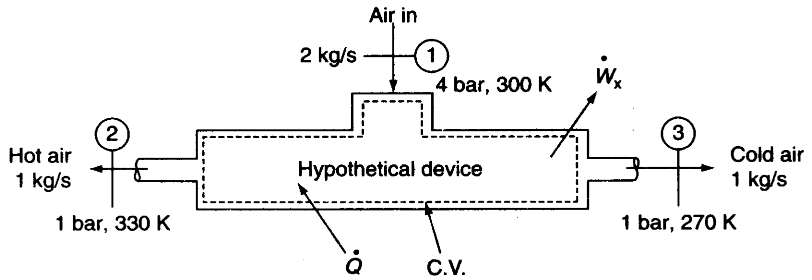
$$(\Delta s)_{\text{univ}} = -0.047 \text{ kJ/kg K}$$

This is impossible. So the flow must be from B to A.

Example 7.9

A hypothetical device is supplied with 2 kg/s of air at 4 bar, 300 K. Two separate streams of air leave the device, as shown in figure below. Each stream is at an ambient pressure of 1 bar, and the mass flow rate is the same for both streams. One of the exit streams is said to be at 330 K while the other is at 270 K. The ambient temperature is at 300 K. Determine whether such a device is possible.

Solution The entropy generation rate for the control volume (Fig. Ex. 7.9) is



[redacted]

$$\dot{S}_{gen} = \sum \dot{m}_e s_e - \sum \dot{m}_i s_i$$

$$= \dot{m}_2 s_2 + \dot{m}_3 s_3 - \dot{m}_1 s_1$$

$$= \dot{m}_2 s_2 + \dot{m}_3 s_3 - (\dot{m}_2 + \dot{m}_3) s_1$$

$$= \dot{m}_2 (s_2 - s_1) + \dot{m}_3 (s_3 - s_1)$$

Now,

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

$$= 1.005 \ln \frac{330}{300} - 0.287 \ln \frac{1}{4}$$

$$= 0.494 \text{ kJ/kg K}$$

$$s_3 - s_1 = c_p \ln \frac{T_3}{T_1} - R \ln \frac{p_3}{p_1}$$

$$= 1.005 \ln \frac{270}{300} - 0.287 \ln \frac{1}{4} = 0.292 \text{ kJ/kgK}$$

$$\dot{S}_{gen} = 1 \times 0.494 + 1 \times 0.292 = 0.786 \text{ kW/K}$$

Since $\dot{S}_{gen} > 0$, the device is possible. Such devices actually exist and are called *vortex tubes*. Although they have low efficiencies, they are suitable for certain applications like rapid cooling of soldered parts, electronic component cooling, cooling of machining operations and so on. The vortex tube is essentially a passive device with no moving parts. It is relatively maintenance free and durable.

Example 7.10

A room is maintained at 27°C *while the surroundings are at* 2°C. *The temperatures of the inner and outer surfaces of the wall* (k = 0.71 *W/mK*) *are measured to be* 21°C *and* 6°C, *respectively. Heat flows steadily through the wall* 5m × 7 *m in cross-section and* 0.32 *m in thickness. Determine* (*a*) *the rate of heat transfer through the wall,* (*b*) *the rate of entropy generation in the wall, and* (*c*) *the rate of total entropy generation with this heat transfer process.*

Solution

$$\dot{Q} = kA\,\frac{\Delta T}{L} = 0.71\ \frac{\text{W}}{\text{mK}} \times (5 \times 7)\ \text{m}^2 \times \frac{(21-6)\text{K}}{0.32\text{m}}$$

$$= 1164.84\ \text{W} \qquad \textit{Ans.}\ \text{(a)}$$

Taking the wall as the system, the entropy balance in rate form gives:

$$\frac{\text{d}S_{\text{wall}}}{\text{d}t} = \dot{S}_{\text{transfer}} + \dot{S}_{\text{gen.wall}}$$

$$0 = \sum \frac{\dot{Q}}{T} + \dot{S}_{\text{gen.wall}}$$

$$0 = \frac{1164.84}{294} - \frac{1164.84}{279} + \dot{S}_{\text{gen.wall}}$$

Rate of entropy generation in the wall

$$\dot{S}_{\text{gen.wall}} = 4.175 - 3.962 = 0.213\ \text{W/K} \qquad \textit{Ans.}\ \text{(b)}$$

The entropy change of the wall is zero during this process, since the state and hence the entropy of the wall does not change anywhere in the wall.

To determine the rate of total entropy generation during this heat transfer process, we extend the system to include the regions on both sides of the wall,

$$\frac{\text{d}S_{\text{wall}}}{\text{d}t} = \dot{S}_{\text{transfer}} + \dot{S}_{\text{gen.total}}$$

$$0 = \sum \frac{\dot{Q}}{T} + \dot{S}_{\text{gen.total}}$$

$$0 = \frac{1164.84}{300} - \frac{1164.84}{275} + \dot{S}_{\text{gen.total}}$$

$$\dot{S}_{\text{gen.total}} = 4.236 - 3.883 = 0.353\ \text{W/K} \qquad \textit{Ans.}\ \text{(c)}$$

Review Questions

7.1 Show that through one point there can pass only one reversible adiabatic.

7.2 State and prove Clausius' theorem.

7.3 Show that entropy is a property of a system.

7.4 How is the entropy change of a reversible process estimated? Will it be different for an irreversible process between the same end states?

7.5 Why is the Carnot cycle on T–s plot a rectangle?

7.6 Establish the inequality of Clausius.

7.7 Give the criteria of reversibility, irreversibility and impossibility of a thermodynamic cycle.

7.8 What do you understand by the entropy principle?

7.9 When the system is at equilibrium, why would any conceivable change in entropy be zero?

7.10 Why is the entropy increase of an isolated system a measure of the extent of irreversibility of the process undergone by the system?

7.11 How did Rudolf Clausius summarize the first and second laws of thermodynamics?

7.12 Show that the transfer of heat through a finite temperature difference is irreversible.

7.13 Show that the adiabatic mixing of two fluids is irreversible.

7.14 What is the maximum work obtainable from two finite bodies at temperatures T_1 and T_2?

7.15 Determine the maximum work obtainable by using one finite body at temperature T and a thermal energy reservoir at temperature T_0, $T > T_0$.

7.16 What are the causes of entropy increase?

7.17 Why is an isentropic process not necessarily an adiabatic process?

7.18 What is the reversible adiabatic work for a steady flow system when K.E. and P.E. changes ae negligibly small? How is it different from that for a closed stationary system?

7.19 Under what conditions is the work done equal to (a) $\int pdv$, (b) $-\int vdp$?

7.20 Why are the equations

$$TdS = dU + pdV$$
$$TdS = dH - Vdp$$

valid for any process between two equilibrium end states?

7.21 Why is the second law called a directional law of nature?

7.22 How is entropy related to molecular disorder in a system?

7.23 Show that entropy varies logarithmically with the disorder number.

7.24 What do you understand by perfect order?

7.25 Give the Nernst-Simon statement of the third law of thermodynamics.

7.26 Why does entropy remain constant in a reversible adiabatic process?

7.27 What do you understand by 'lost work'?

7.28 The amount of entropy generation quantifies the intrinsic irreversibility of a process. Explain.

7.29 Show that S_{gen} is not a thermodynamic property.

7.30 Give the expression for the entropy generation rate for a control volume of a steady flow system.

7.31 What is the entropy generation in the isothermal dissipation of work?

7.32 What is the entropy generation in the adiabatic dissipation of work?

7.33 What do you understand by entropy transfer? Why is entropy transfer associated with heat transfer and not with work transfer?

Problems

7.1 On the basis of the first law fill in the blank spaces in the following table of imaginary heat engine cycles. On the basis of the second law classify each cycle as reversible, irreversible, or impossible.

Cycle Temperature		*Rate of Heat Flow*		*Rate of work*	*Efficiency*
Source	*Sink*	*Supply*	*Rejection*	*Output*	
(a) 327°C	27°C	420 kJ/s	230 kJ/s	...kW	
(b) 1000°C	100°C	...kJ/min	4.2 MJ/min	...kW	65%
(c) 750 K	300 K	... kJ/s	... kJ/s	26 kW	60%
(d) 700 K	300 K	3 kW	... kW	2 kW	—

Ans. (a) Irreversible, (b) Irreversible, (c) Reversible, (d) Impossible

7.2 The latent heat of fusion of water at 0°C is 335 kJ/kg. How much does the entropy of 1 kg of ice change as it melts into water in each of the following ways: (a) Heat is supplied reversibly to a mixture of ice and water at 0°C. (b) A mixture of ice and water at 0°C is stirred by a paddle wheel.
Ans. 1.2271 kJ/K

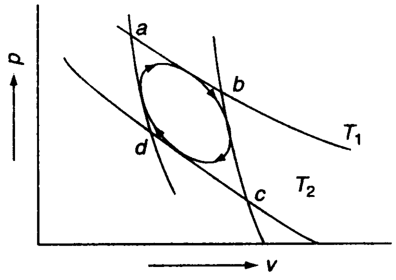
7.3 Twokgofwaterat80°Caremixedadiabaticallywith 3kgofwaterat30°Cinaconstantpressureprocessof 1 atmosphere. Find the increase in the entropy of the total mass of water due to the mixing process (c_p of water = 4.187 kJ/kg K). *Ans.* 0.0592 kJ/K

7.4 In a Carnot cycle, heat is supplied at 350°C and rejected at 27°C. The working fluid is water which, while receiving heat, evaporates from liquid at 350°C to steam at 350°C. The associated entropy change is 1.44 kJ/kg K. (a) If the cycle operates on a stationary mass of 1 kg of water, how much is the work done per cycle, and how much is the heat supplied? (b) If the cycle operates in steady flow with a power output of 20 kW, what is the steam flow rate?
Ans. (a) 465.12, 897.12 kJ/kg, (b) 0.043 kg/s

7.5 A heat engine receives reversibly 420 kJ/cycle of heat from a source at 327°C, and rejects heat reversibly to sink at 27°C. There are no other heat transfers. For each of the three hypothetical amounts of heat rejected, in (a), (b), and (c) below, compute the cyclic integral of $\oint dQ/T$. From these results show which case is irreversible, which reversible, and which impossible: (a) 210 kJ/cycle rejected, (b) 105 kJ/cycle rejected, (c) 315 kJ/cycle rejected.

Ans. (a) Reversible, (b) Impossible, (c) Irreversible

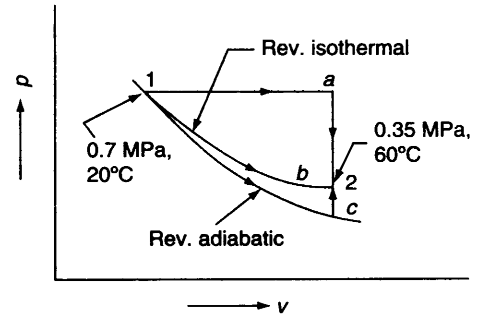
7.6 In Fig. P. 7.6, *abcd* represents a Carnot cycle bounded by two reversible adiabatics and two reversible isotherms at temperatures T_1 and T_2 ($T_1 > T_2$). The oval figure is a reversible cycle, where heat is absorbed at temperature less than, or equal to, T_1, and rejected at temperatures greater than, or equal to, T_2. Prove that the efficiency of the oval cycle is less than that of the Carnot cycle.



7.7 Water is heated at a constant pressure of 0.7 MPa. The boiling point is 164.97°C. The initial temperature of water is 0°C. The latent heat of evaporation is 2066.3 kJ/kg. Find the increase of entropy of water, if the final state is steam.

Ans. 6.6967 kJ/kg K

7.8 One kg of air initially at 0.7 MPa, 20°C changes to 0.35 MPa, 60°C by the three reversible non-flow processes, as shown in Fig. P. 7.8. Process 1-*a*-2 consists of a constant pressure expansion followed by a constant volume cooling, process 1-*b*-2 an isothermal expansion followed by a constant pressure expansion, and process 1-*c*-2 an adiabatic expansion followed by a constant volume heating. Determine the changes of internal energy, enthalpy, and entropy for each process, and find the work transfer and heat transfer for each process. Take $c_p = 1.005$ and $c_v = 0.718$ kJ/kg, K and assume the specific heats to be constant. Also assume for air $pv = 0.287$ T, where p is the pressure in kPa, v the specific volume in m^3/kg, and T the temperature in K.



7.9 Ten grammes of water at 20°C is converted into ice at –10°C at constant atmospheric pressure. Assuming the specific heat of liquid water to remain constant at 4.2 J/gK and that of ice to be half of this value, and taking the latent heat of fusion of ice at 0°C to be 335 J/g, calculate the total entropy change of the system.

Ans. 16.02 J/K

7.10 Calculate the entropy change of the universe as a result of the following processes:

(a) A copper block of 600 g mass and with C_p of 150 J/K at 100°C is placed in a lake at 8°C.
(b) The same block, at 8°C, is dropped from a height of 100 m into the lake.
(c) Two such blocks, at 100 and 0°C, are joined together.

Ans. (a) 6.63 J/K, (b) 2.095 J/K, (c) 3.64 J/K

7.11 A system maintained at constant volume is initially at temperature T_1, and a heat reservoir at the lower temperature T_0 is available. Show that the maximum work recoverable as the system is cooled to T_0 is

$$W = C_v\left[(T_1 - T_0) - T_0 \ln \frac{T_1}{T_0}\right]$$

7.12 A body of finite mass is originally at temperature T_1 which is higher than that of a reservoir at temperature T_2. Suppose an engine operates in a cycle between the body and the reservoir until it lowers the temperature of the body from T_1 to T_2, thus extracting heat Q from the body. If the engine does work W, then it will reject heat $Q-W$ to the reservoir at T_2. Applying the entropy principle, prove that the maximum work obtainable from the engine is

$$W_{(\max)} = Q - T_2 (S_1 - S_2)$$

where $S_1 - S_2$ is the entropy decrease of the body. If the body is maintained at constant volume having constant volume heat capacity $C_v = 8.4$ kJ/K which is independent of temperature, and if $T_1 = 373$ K and $T_2 = 303$ K, determine the maximum work obtainable.

Ans. 58.96 kJ

7.13 Each of three identical bodies satisfies the equation $U = CT$, where C is the heat capacity of each of the bodies. Their initial temperatures are 200 K, 250 K, and 540 K. If $C = 8.4$ kJ/K, what is the maximum amount of work that can be extracted in a process in which these bodies are brought to a final common temperature?

Ans. 756 kJ

7.14 In the temperature range between 0°C and 100°C a particular system maintained at constant volume has a heat capacity.

$$C_V = A + 2BT$$

with $A = 0.014$ J/K and $B = 4.2 \times 10^{-4}$ J/K^2.

A heat reservoir at 0°C and a reversible work source are available. What is the maximum amount of work that can be transferred to the reversible work source as the system is cooled from 100°C to the temperature of the reservoir?

Ans. 4.508 J

7.15 A reversible engine, as shown in Fig. P. 7.15 during a cycle of operation draws 5 MJ from the 400 K reservoir and does 840 kJ of work. Find the amount and direction of heat interaction with other reservoirs.

Ans. $Q_2 = +4.98$ MJ, $Q_3 = -0.82$ MJ

7.16 For a fluid for which pv/T is a constant quantity equal to R, show that the change in specific entropy between two states A and B is given by

$$s_B - s_A = \int_{T_A}^{T_B} \frac{C_p}{T}\, dT - R \ln \frac{p_B}{p_A}$$

A fluid for which R is a constant and equal to 0.287 kJ/kg K, flows steadily through an adiabatic machine, entering and leaving through two adiabatic pipes. In one of these pipes the pressure and temperature are 5 bar and 450 K and in the other pipe the pressure and temperature are 1 bar and 300 K respectively. Determine which pressure and temperature refer to the inlet pipe. For the given temperature range, c_p is given by

$$c_p = a \ln T + b$$

where T is the numerical value of the absolute temperature and $a = 0.026$ kJ/kg K, $b = 0.86$ kJ/kg K.

Ans. $s_B - s_A = 0.0509$ kJ/kg K, A is the inlet pipe.

7.17 Two vessels, A and B, each of volume 3 m^3 may be connected by a tube of negligible volume. Vessel A contains air at 0.7 MPa, 95°C, while vessel B contains air at 0.35 MPa, 205°C. Find the change of entropy when A is connected to B by working from the first principles and assuming the mixing to be complete and adiabatic. For air take the relations as given in Example 7.8. *Ans.* 0.947 kJ/K

7.18 (a) An aluminium block ($c_p = 400$ J/kg K) with a mass of 5 kg is initially at 40°C in room air at 20°C. It is cooled reversibly by transferring heat to a completely reversible cyclic heat engine until the block reaches 20°C. The 20°C room air serves as a constant temperature sink for the engine. Compute (i) the change in entropy for the block, (ii) the change in entropy for the room air, (iii) the work done by the engine.

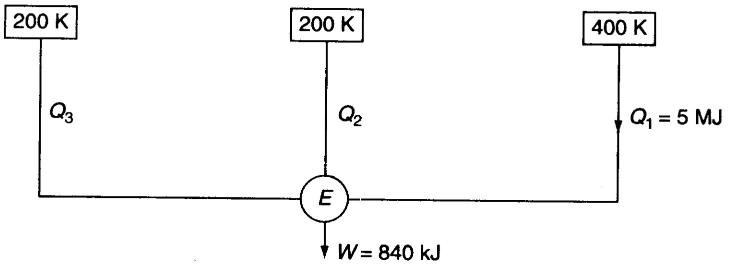


Fig. P. 7.15

(b) If the aluminium block is allowed to cool by natural convection to room air, compute (i) the change in entropy for the block, (ii) the change in entropy for the room air (iii) the net change in entropy for the universe. *Ans.* (a) – 134.2 J/K, + 132 J/K, 1306 J, (b) – 132 J/K, + 136.5 J/K, 42.5 J/K

7.19 Two bodies of equal heat capacities C and temperatures T_1 and T_2 form an adiabatically closed system. What will the final temperature be if one lets this system come to equilibrium (a) freely? (b) reversibly? (c) What is the maximum work which can be obtained from this system?

7.20 A resistor of 30 ohms is maintained at a constant temperature of 27°C while a current of 10 amperes is allowed to flow for 1 sec. Determine the entropy change of the resistor and the universe.

Ans. $(\Delta S)_{\text{resistor}} = 0$, $(\Delta S)_{\text{univ}} = 10$ J/K

If the resistor initially at 27°C is now insulated and the same current is passed for the same time, determine the entropy change of the resistor and the universe. The specific heat of the resistor is 0.9 kJ/kg K and the mass of the resistor is 10 g.

Ans. $(\Delta S)_{\text{univ}} = 6.72$ J/K

7.21 An adiabatic vessel contains 2 kg of water at 25°C. By paddle-wheel work transfer, the temperature of water is increased to 30°C. If the specific heat of water is assumed constant at 4.187 kJ/kg K, find the entropy change of the universe. *Ans.* 0.139 kJ/K

7.22 A copper rod is of length 1 m and diameter 0.01 m. One end of the rod is at 100°C, and the other at 0°C. The rod is perfectly insulated along its length and the thermal conductivity of copper is 380 W/mK. Calculate the rate of heat transfer along the rod and the rate of entropy production due to irreversibility of this heat transfer. *Ans.* 2.985 W, 0.00293 W/K

7.23 A body of constant heat capacity C_p and at a temperature T_i is put in contact with a reservoir at a higher temperature T_f. The pressure remains constant while the body comes to equilibrium with the reservoir. Show that the entropy change of the universe is equal to

$$C_p\left[\frac{T_i - T_f}{T_f} - \ln\left(1 + \frac{T_i - T_f}{T_f}\right)\right]$$

Prove that this entropy change is positive.

Given:

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots -$$

where $x < 1$.

7.24 An insulated 0.75 kg copper calorimeter can containing 0.2 kg water is in equilibrium at a temperature of 20°C. An experimenter now places 0.05 kg of ice at 0°C in the calorimeter and encloses the latter with a heat insulating shield. (a) When all the ice has melted and equilibrium has been reached, what will be the temperature of water and the can? The specific heat of copper is 0.418 kJ/kg K and the latent heat of fusion of ice is 333 kJ/kg. (b) Compute the entropy increase of the universe resulting from the process. (c) What will be the minimum work needed by a stirrer to bring back the temperature of water to 20°C?

Ans. (a) 4.68°C, (b) 0.00276 kJ/K, (c) 20.84 kJ

7.25 Show that if two bodies of thermal capacities C_1 and C_2 at temperatures T_1 and T_2 are brought to the same temperature T by means of a reversible heat engine, then

$$\ln T = \frac{C_1 \ln T_1 + C_2 \ln T_2}{C_1 + C_2}$$

7.26 Two blocks of metal, each having a mass of 10 kg and a specific heat of 0.4 kJ/kg K, are at a temperature of 40°C. A reversible refrigerator receives heat from one block and rejects heat to the other. Calculate the work required to cause a temperature difference of 100°C between the two blocks. *Ans.* 32 kJ

7.27 A block of iron weighing 100 kg and having a temperature of 100°C is immersed in 50 kg of water at a temperature of 20°C. What will be the change of entropy of the combined system of iron and water? Specific heats of iron and water are 0.45 and 4.18 kJ/kg K respectively.*Ans.* 1.24 kJ/K

7.28 36 g of water at 30°C are converted into steam at 250°C at constant atmospheric pressure. The specific heat of water is assumed constant at 4.2 J/gK and the latent heat of vaporization at 100°C is 2260 J/g. For water vapour, assume $pV = mRT$ where $R = 0.4619$ kJ/kg K, and

$\frac{c_p}{R} = a + bT + cT^2$, where $a = 3.634$,

$b = 1.195 \times 10^{-3}\ \text{K}^{-1}$ and $c = 0.135 \times 10^{-6}\ \text{K}^{-2}$

Calculate the entropy change of the system.

Ans. 273.1 J/K

7.29 A 50 ohm resistor carrying a constant current of 1 A is kept at a constant temperature of 27°C by a stream of cooling water. In a time interval of 1 s, (a) what is the change in entropy of the resistor? (b) What is the change in entropy of the universe?
Ans. (a) 0, (b) 0.167 J/K

7.30 A lump of ice with a mass of 1.5 kg at an initial temperature of 260 K melts at the pressure of 1 bar as a result of heat transfer from the environment. After some time has elapsed the resulting water attains the temperature of the environment, 293 K. Calculate the entropy production associated with this process. The latent heat of fusion of ice is 333.4 kJ/kg, the specific heats of ice and water are 2.07 and 4.2 kJ/kg K respectively, and ice melts at 273.15 K. *Ans.* 0.1514 kJ/K

7.31 An ideal gas is compressed reversibly and adiabatically from state *a* to state *b*. It is then heated reversibly at constant volume to state *c*. After expanding reversibly and adiabatically to state *d* such that $T_b = T_d$, the gas is again reversibly heated at constant pressure to state *e* such that $T_e = T_c$. Heat is then rejected reversibly from the gas at constant volume till it returns to state *a*. Express T_a in terms of T_b and T_c. If $T_b = 555$ K and $T_c = 835$ K, estimate T_a. Take $\gamma = 1.4$.
Ans. $T_a = \dfrac{T_b^{\gamma+1}}{T_c^{\gamma}}$, 313.29 K

7.32 Liquid water of mass 10 kg and temperature 20°C is mixed with 2 kg of ice at – 5°C till equilibrium is reached at 1 atm pressure. Find the entropy change of the system. Given: c_p of water = 4.18 kJ/kg K, c_p of ice = 2.09 kJ/kg K and latent heat of fusion of ice = 334 kJ/kg. *Ans.* 104.9 J/K

7.33 A thermally insulated 50-ohm resistor carries a current of 1 A for 1 s. The initial temperature of the resistor is 10°C. Its mass is 5 g and its specific heat is 0.85 J/g K. (a) What is the change in entropy of the resistor? (b) What is the change in entropy of the universe?
Ans. (a) 0.173 J/K (b) 0.173 J/K

7.34 The value of c_p for a certain substance can be represented by $c_p = a + bT$. (a) Determine the heat absorbed and the increase in entropy of a mass *m* of the substance when its temperature is increased at constant pressure from T_1 to T_2. (b) Find the increase in the molal specific entropy of copper, when the temperature is increased at constant pressure from 500 to 1200 K. Given for copper: when $T = 500$ K, $c_p = 25.2 \times 10^3$ and when $T = 1200$ K, $c_p = 30.1 \times 10^3$ J/k mol K.
Ans. (a) *m*

$$\left[a\left(T_2 - T_1\right) + \frac{b}{2}\left(T_2^2 - T_1^2\right)\right] m \left[a \ln \frac{T_2}{T_1} + b\left(T_2 - T_1\right)\right]$$

(b) 23.9 kJ/k mol K

7.35 An iron block of unknown mass at 85°C is dropped into an insulated tank that contains 0.1m³ of water at 20°C. At the same time a paddle-wheel driven by a 200 W motor is activated to stir the water. Thermal equilibrium is established after 20 min when the final temperature is 24°C. Determine the mass of the iron block and the entropy generated during the process. *Ans.* 52.2 kg, 1.285 kJ/K

7.36 A piston-cylinder device contains 1.2 kg of nitrogen gas at 120 kPa and 27°C. The gas is now compressed slowly in a polytropic process during which $pV^{1.3}$ = constant. The process ends when the volume is reduced by one-half. Determine the entropy change of nitrogen during this process. *Ans.* – 0.0615 kJ/K.

7.37 Air enters a compressor at ambient conditions of 96 kPa and 17°C with a low velocity and exits at 1 MPa, 327°C, and 120 m/s. The compressor is cooled by the ambient air at 17°C at a rate of 1500 kJ/min. The power input to the compressor is 300 kW. Determine (a) the mass flow rate of air and (b) the rate of entropy generation.
Ans. (a) 0.851 kg/s, (b) 0.144 kW/K

7.38 A gearbox operating at steady state receives 0.1 kW along the input shaft and delivers 0.095 kW along the output shaft. The outer surface of the gearbox is at 50°C. For the gearbox, determine (a) the rate of heat transfer, (b) the rate at which entropy is produced. *Ans.* (a) – 0.005 kW, (b) 1.54×10^{-5} kW/K

7.39 At steady state, an electric motor develops power along its output shaft at the rate of 2 kW while drawing 20 amperes at 120 volts. The outer surface of the motor is at 50°C. For the motor, determine the rate of heat transfer and the rate of entropy generation. *Ans.* – 0.4 kW, 1.24×10^{-3} kW/K

7.40 Show that the minimum theoretical work input required by a refrigeration cycle to bring two finite bodies from the same initial temperature to the final temperatures of T_1 and T_2 ($T_2 < T_1$) is given by

$$W_{\min} = mc\,[2(T_1T_2)^{1/2} - T_1 - T_2]$$

7.41 A rigid tank contains an ideal gas at 40°C that is being stirred by a paddle wheel. The paddle wheel does 200 kJ of work on the ideal gas. It is observed that the temperature of the ideal gas remains constant during this process as a result of heat transfer between the system and the surroundings at 25°C. Determine (a) the entropy change of the ideal gas and (b) the total entropy generation.

Ans. (a) 0, (b) 0.671 kJ/K

7.42 A cylindrical rod of length L insulated on its lateral surface is initially in contact at one end with a wall at temperature T_1 and at the other end with a wall at a lower temperature T_2. The temperature within the rod initially varies linearly with position x according to:

$$T(x) = T_1 - \frac{T_1 - T_2}{L}x$$

The rod is insulated on its ends and eventually comes to a final equilibrium state where the temperature is T_f. Evaluate T_f and in terms of T_1 and T_2, and show that the amount of entropy generated is:

$$S_{gen} = mc\left[1 + \ln T_f + \frac{T_2}{T_1 - T_2}\ln T_2 - \frac{T_1}{T_1 - T_2}\ln T_1\right]$$

where c is the specific heat of the rod.

Ans. $T_f = [T_1 + T_2]/2$

7.43 Air flowing through a horizontal, insulated duct was studied by students in a laboratory. One student group measured the pressure, temperature, and velocity at a location in the duct as 0.95 bar, 67°C, 75 m/s. At another location the respective values were found to be 0.8 bar, 22°C, 310 m/s. The group neglected to note the direction of flow, however. Using the known data, determine the direction. *Ans.* Flow is from right to left

7.44 Nitrogen gas at 6 bar, 21°C enters an insulated control volume operating at steady state for which $W_{C.V.} = 0$. Half of the nitrogen exits the device at 1 bar, 82°C and the other half exits at 1 bar, – 40°C. The effects of KE and PE are negligible. Employing the ideal gas model, decide whether the device can operate as described.

Ans. Yes, the device can operate as described

C H A P T E R

8

Available Energy, Exergy and Irreversibility

8.1 AVAILABLE ENERGY

The sources of energy can be divided into two groups, viz. high grade energy and low grade energy. The conversion of high grade energy to shaft work is exempted from the limitations of the second law, while conversion of low grade energy is subject to them.

The examples of two kinds of energy are:

High grade energy	*Low grade energy*
(a) Mechanical work	(a) Heat or thermal energy
(b) Electrical energy	(b) Heat derived from nuclear fission or fusion
(c) Water power	(c) Heat derived from combustion of fossil fuels
(d) Wind power	
(e) Kinetic energy of a jet	
(f) Tidal power	

The bulk of the high grade energy in the form of mechanical work or electrical energy is obtained from sources of low grade energy, such as fuels, through the medium of the cyclic heat engine. The complete conversion of low grade energy, heat, into high grade energy, shaft-work, is impossible by virtue of the second law of thermodynamics. That part of the low grade energy which is available for conversion is referred to as available energy, while the part which, according to the second law, must be rejected, is known as *unavailable energy.*

Josiah Willard Gibbs is accredited with being the originator of the availability concept. He indicated that environment plays an important part in evaluating the available energy.

8.2 AVAILABLE ENERGY REFERRED TO A CYCLE

The maximum work output obtainable from a certain heat input in a cyclic heat engine (Fig. 8.1) is called the *available energy* (A.E.), or the available part of the energy supplied. The minimum energy that has to be rejected to the sink by the second law is called the *unavailable energy* (U.E.), or the unavailable part of the energy supplied.

Therefore, $$Q_1 = \text{A.E.} + \text{U.E.} \tag{8.1}$$

or $$W_{\text{max}} = \text{A.E.} = Q_1 - \text{U.E.}$$

For the given T_1 and T_2,

$$\eta_{\text{rev}} = 1 - \frac{T_2}{T_1}$$

For a given T_1, η_{rev} will increase with the decrease of T_2. The lowest practicable temperature of heat rejection is the *temperature of the surroundings*, T_0

$$\therefore \quad \eta_{\text{max}} = 1 - \frac{T_0}{T_1}$$

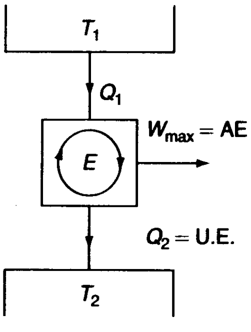


Fig. 8.1 *Available and unavailable energy in a cycle*

and $$W_{\max} = \left(1 - \frac{T_0}{T_1}\right) Q_1$$

Let us consider a finite process x–y, in which heat is supplied reversibly to a heat engine (Fig. 8.2). Taking an elementary cycle, if $\text{đ}Q_1$ is the heat received by the engine reversibly at T_1, then

$$\text{đ}W_{\max} = \frac{T_1 - T_0}{T_1}\text{đ}Q_1 = \text{đ}Q_1 - \frac{T_0}{T_1}\text{đ}Q_1 = \text{A.E.}$$

For the heat engine receiving heat for the whole process x–y, and rejecting heat at T_0

$$\int_x^y \text{đ}W_{\max} = \int_x^y \text{đ}Q_1 - \int_x^y \frac{T_0}{T_1}\text{đ}Q_1$$

$$\therefore \quad W_{\max} = \text{A.E.} = Q_{xy} - T_0(s_y - s_x) \qquad (8.2)$$

$$\text{or} \quad \text{U.E.} = Q_{xy} - W_{\max}$$

$$\text{or} \quad \text{U.E.} = T_0(s_y - s_x)$$

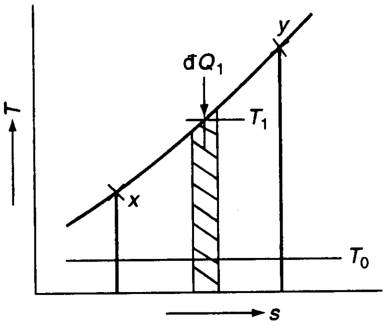


Fig. 8.2 *Availability of energy*

The unavailable energy is thus the product of the lowest temperature of heat rejection, and the change of entropy of the system during the process of supplying heat (Fig. 8.3). The available energy is also known as *exergy* and the unavailable energy as *aenergy*, the words first coined by Rant (1956).

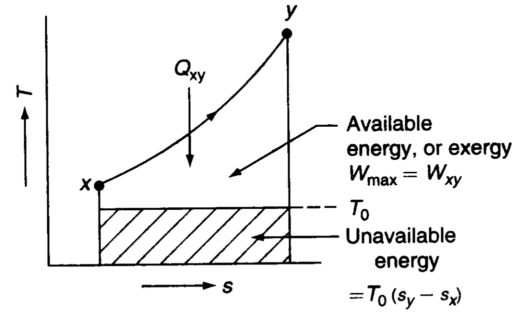


Fig. 8.3 *Unavailable energy by the second law*

8.2.1 Decrease in Available Energy when Heat is Transferred through a Finite Temperature Difference

Whenever heat is transferred through a finite temperature difference, there is a decrease in the availability of energy so transferred.

Let us consider a reversible heat engine operating between T_1 and T_0 (Fig. 8.4). Then

$$Q_1 = T_1\,\Delta s,\; Q_2 = T_0\,\Delta s, \text{ and } W = \text{A.E.} = (T_1 - T_0)\,\Delta s$$

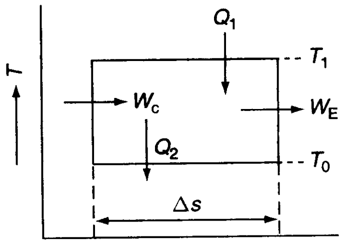


Fig. 8.4 *Carnot cycle*

Let us now assume that heat Q_1 is transferred through a finite temperature difference from the reservoir or source at T_1 to the engine absorbing heat at T'_1, lower than T_1 (Fig. 8.5). The availability of Q_1 as received by the engine at T'_1 lower than T_1 (Fig. 8.5). The availability of Q_1 as received by the engine at T'_1 can be found by allowing the engine to operate reversibly in a cycle between T'_1 and T_0, receiving Q_1 and rejecting Q_2.

Now $\quad Q_1 = T_1\,\Delta s = T'_1\,\Delta s'$

Since $\quad T_1 > T'_1, \therefore \Delta s' > \Delta s \quad Q_2 = T_0\,\Delta s \quad Q'_2 = T_0\,\Delta s'$

Since $\quad \Delta s' > \Delta s \quad \therefore Q'_2 > Q_2$

$\therefore \quad W' = Q_1 - Q'_2 = T'_1\,\Delta s' - T_0\,\Delta s'$

and $\quad W = Q_1 - Q_2 = T'_1\,\Delta s - T_0\,\Delta s$

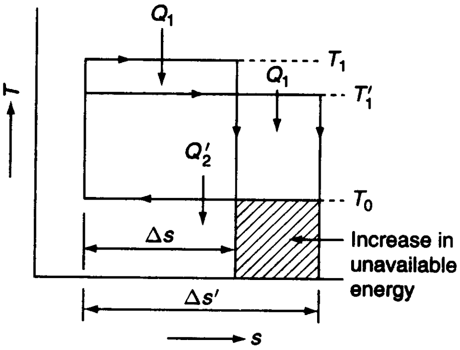
$\therefore \quad W' < W$, because $Q'_2 > Q_2$

Available energy or exergy lost due to irreversible heat transfer through finite temperature difference between the source and the working fluid during the heat addition process is given by $W - W' = Q'_2 - Q_2 = T_0 (\Delta s' - \Delta s)$

or, decrease in A.E. $= T_0 (\Delta s' - \Delta s)$

The decrease in available energy or exergy is thus the product of the lowest feasible temperature of heat rejection and the additional entropy change in the system while receiving heat irreversibly, compared to the case of reversible heat transfer from the same source.

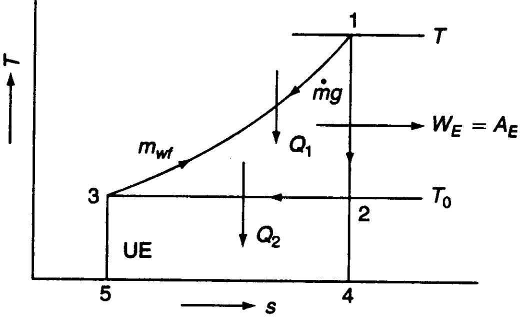
The greater is the temperature difference $(T_1 - T'_1)$, the greater is the heat rejection Q'_2 and the greater will be the unavailable part of the energy supplied or anergy (Fig. 8.5). Energy is said to be degraded each time it flows through a finite temperature difference.



Increase in unavailable energy due to heat transfer through a finite temperature difference

8.2.2 Available Energy from a Finite Energy Source

Let us consider a hot gas of mass m_g at temperature T when the environmental temperature is T_0 (Fig. 8.6). Let the gas be cooled at constant pressure from state 1 at temperature T to state 3 at temperature T_0 and the heat given up by the gas, Q_1, be utilized in heating up reversibly a working fluid of mass m_{wf} from state 3 to state 1 along the same path so that the temperature difference between the gas and the working fluid at any instant is zero and hence, the entropy increase of the universe is also zero. The working fluid expands reversibly and adiabatically in an engine or turbine from sate 1 to state 2 doing work W_E, and then rejects heat Q_2 reversibly and isothermally to return to the initial state 3 to complete a heat engine cycle.



Available energy of a finite energy source

Here, $Q_1 = m_g c_{p_g} (T - T_0) = m_{wf} c_{pwf} (T - T_0) =$ Area 1–4–5–3–1

$$\therefore \quad m_g c_{p_g} = m_{wf} c_{p_{wf}}$$

$$\Delta S_{gas} = \int_T^{T_0} m_g c_{p_g} \frac{dT}{T} = m_g c_{p_g} \ln \frac{T_0}{T} \text{ (negative)}$$

$$\Delta S_{wf} = \int_{T_0}^{T} m_{wf} c_{p_{wf}} \frac{dT}{T} = m_{wf} c_{p_{wf}} \ln \frac{T}{T_0} \text{ (positive)}$$

$$\therefore \quad \Delta S_{univ} = \Delta S_{gas} + \Delta S_{wf} = 0$$

$$Q_2 = T_0 \Delta S_{wf} = T_0 m_{wf} c_{p_{wf}} \ln \frac{T}{T_0} = \text{Area 2–4–5–3}$$

$$\therefore \quad \text{Available energy} = W_{max}$$

$$= Q_1 - Q_2 = m_g c_{p_g} (T - T_0) - T_0 m_g c_{p_g} \ln \frac{T}{T_0}$$

$$= \text{Area 1–2–3–1}$$

Therefore, the available energy or exergy of a gas of mass m_g at temperature T is given by

$$AE = m_g\, c_{p_g} \left[(T - T_0) - T_0 \ln \frac{T}{T_0}\right] \tag{8.3}$$

This is similar to Eq. (7.22) derived from the entropy principle.

8.3 QUALITY OF ENERGY

Let us assume that a hot gas is flowing through a pipeline (Fig. 8.7). Due to heat loss to the surroundings, the temperature of the gas decreases continuously from inlet at state *a* to the exit at state *b*. Although the process is irreversible, let us assume a reversible isobaric path between the inlet and exit states of the gas (Fig. 8.8). For an infinitesimal reversible process at constant pressure,

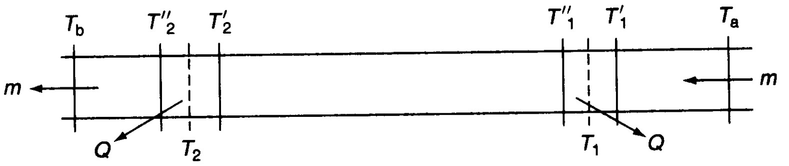


Fig. 8.7 Heat loss from a hot gas flowing through a pipeline

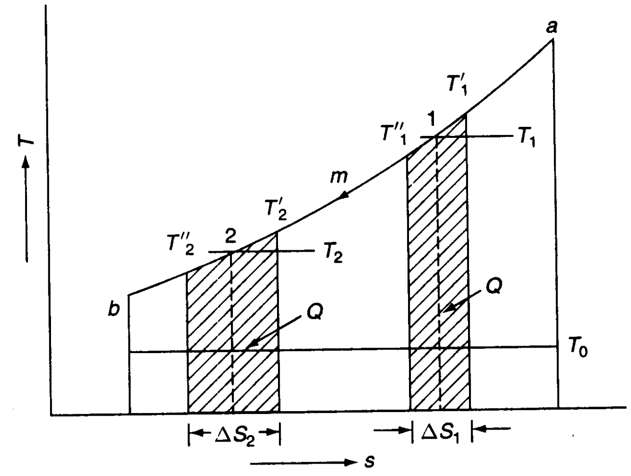


Fig. 8.8 Energy quality at state 1 is superior to that at state 2

$$dS = \frac{mc_p dT}{T}$$

or

$$\frac{dT}{dS} = \frac{T}{mc_p} \tag{8.4}$$

where m is the mass of gas flowing and c_p is its specific heat. The slope dT/dS depends on the gas temperature T. As T increases, the slope increases, and if T decreases the slope decreases.

Let us assume that Q units of heat are lost to the surroundings as the temperature of the gas decreases from T'_1 to T''_1, T_1 being the average of the two. Then,

Heat loss $$Q = mc_p (T'_1 - T''_1) = T_1\, \Delta S_1 \tag{8.5}$$

Exergy lost with this heat loss at temperature T_1 is

$$W_1 = Q - T_0\,\Delta S_1 \tag{8.6}$$

When the gas temperature has reached T_2 ($T_2 < T_1$), let us assume that the same heat loss Q occurs as the gas temperature decreases from T'_2 to T''_2, T_2 being the average temperature of the gas. Then

Heat loss
$$Q = mc_p\,(T'_2 - T''_2) = T_2\,\Delta S_2 \tag{8.7}$$

Exergy lost with this heat loss at temperature T_2 is

$$W_2 = Q - T_0\,\Delta S_2 \tag{8.8}$$

From Eqs (8.5) and (8.7), since $T_1 > T_2$

$$\Delta S_1 < \Delta S_2$$

Therefore, from Eqs (8.6) and (8.8),

$$W_1 > W_2 \tag{8.9}$$

The loss of exergy is more, when heat loss occurs at a higher temperature T_1 than when the same heat loss occurs at a lower temperature T_2. Therefore, a heat loss of 1 kJ at, say, 1000°C is more harmful than the same heat loss of 1 kJ at, say, 100°C. Adequate insulation must be provided for high temperature fluids ($T >> T_0$) to prevent the precious heat loss. This many not be so important for low temperature fluids ($T \sim T_0$), since the loss of available energy from such fluids would be low. (Similarly, insulation must be provided adequately for very low temperature fluids ($T << T_0$) to prevent heat gain from surroundings and preserve available energy.)

The available energy or exergy of a fluid at a higher temperature T_1 is more than that at a lower temperature T_2, and decreases as the temperature decreases. When the fluid reaches the ambient temperature, its exergy is zero.

The second law, therefore, affixes a quality to energy of a system at any state. The quality of energy of a gas at, say, 1000°C is superior to that at, say, 100°C, since the gas at 1000°C has the capacity of doing more work than the gas at 100°C, under the same environmental conditions. An awareness of this energy quality as of energy quantity is essential for the efficient use of our energy resources and for energy conservation. The concept of *available energy or exergy provides a useful measure of this energy quality.*

8.3.1 Law of Degradation of Energy

The available energy of a system decreases as its temperature or pressure decreases and approaches that of the surroundings. When heat is transferred from a system, its temperature decreases and hence the quality of its energy deteriorates. The degradation is more for energy loss at a higher temperature than that at a lower temperature. Quantity-wise the energy loss may be the same, but quality-wise the losses are different. *While the first law states that energy is always conserved quantity-wise, the second law emphasizes that energy always degrades quality-wise.* When a gas is throttled adiabatically from a high to a low pressure, the enthalpy (or energy per unit mass) remains the same, but there is a degradation of energy or available work. The same holds good for pressure drop due to friction of a fluid flowing through an insulated pipe. If the first law is the law of conservation of energy, the second law is called the *law of degradation of energy*. Energy is always conserved, but its quality is always degraded.

Article 8.2.1 which shows how energy gets degraded by thermal irreversibility and produces less useful work can be explained in little different way. Let two bodies 1 and 2 of constant heat capacities C_1 and C_2 be at temperatures T_1 and T_2 ($T_1 > T_2$). These are connected by a rod and a small quantity of head Q flows from 1 to 2. The total change of entropy is:

$$\Delta S = \Delta S_1 + \Delta S_2 = Q\left[\frac{1}{T_2} - \frac{1}{T_1}\right] > 0 \quad (\text{since } T_1 > T_2)$$

The entropy will continue to increase till thermal equilibrium is reached.

Let us now suppose that instead of allowing heat Q to flow from 1 to 2, we used it to operate a Carnot engine and obtain mechanical work, with T_0 as the sink temperature. The maximum work obtainable is:

$$W_1 = Q\left[1-\frac{T_0}{T_1}\right]$$

If, however, we first allow Q to flow from 1 to 2 and then use it to operate the Carnot engine, we obtain:

$$W_2 = Q\left[1-\frac{T_0}{T_2}\right] < W_1$$

Thus, in the course of the irreversible heat conduction the energy has become degraded to the extent that the useful work has been decreased by

$$\Delta W = W_1 - W_2 = T_0 \Delta S$$

The increase in entropy in an irreversible change is thus a measure of the extent to which energy becomes degraded in that change. Conversely, in order to extract the maximum work from a system, changes must be performed in a reversible manner so that total entropy ($\Delta S_{sys} + \Delta S_{sur}$) is conserved.

It is worth pointing that if the two bodies were allowed to reach thermal equilibrium (a) by heat conduction and (b) by operating a Carnot engine between them and extracting work, the final equilibrium temperatures would be different in the two cases. In the first, $U_1 + U_2$ is conserved and the final temperature is:

$$T_f^{(U)} = \frac{C_1T_1 + C_2T_2}{C_1 + C_2}$$

In the second case, $S_1 + S_2$ is conserved and W= $-$ $\Delta U\,(-\Delta U_1 + U_2)$ ($\because$ $dW = Q - dU, < Tds - dU$ so that $dW_{max} = -dU$). In the isentropic process, the final temperature is given by:

$$T_f^{(s)} = T_1^{C_1/(C_1+C_2)}\, T_2^{C_2/(C_1+C_2)} < T_f^{(U)}$$

If $$C_1 = C_2 = C, T_f^{(U)} = \frac{T_1+T_2}{2} \text{ and } T_f^{(s)} = (T_1T_2)^{1/2}$$

The difference in final temperature is due to the lower value of the total internal energy which results from work having been done at the expense of internal energy (see Art 7.9.3).

Similarly, it can be shown that due to mechanical irreversibility also, energy gets degraded so that the degradation of energy quality is a universal principle.

8.4 MAXIMUM WORK IN A REVERSIBLE PROCESS

Let us consider a closed stationary system undergoing a reversible process R from state 1 to state 2 interacting with the surroundings at p_0, T_0 (Fig. 8.9). Then by the first law,

$$Q_R = U_2 - U_1 + W_R \qquad (8.10)$$

If the process were irreversible, as represented by the dotted line I, connecting the same equilibrium end states,

$$Q_I = U_2 - U_1 + W_I \qquad (8.11)$$

Therefore, from Eqs (8.10) and (8.11),

$$Q_R - Q_I = W_R - W_I \qquad (8.12)$$

Now,

$$\Delta S_{sys} = S_2 - S_1$$

and $$\Delta S_{surr} = -\frac{Q}{T_0}$$

By the second law, $$\Delta S_{univ} \geq 0$$

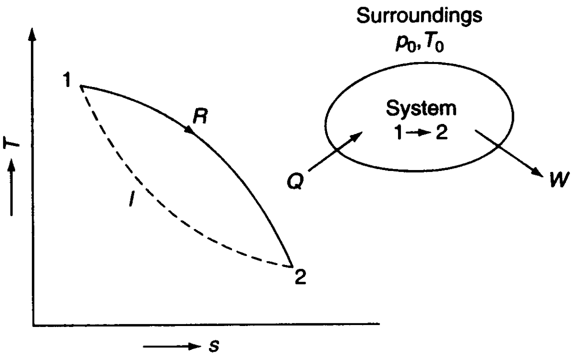


Fig. 8.9 Maximum work done by a closed system

For a reversible process, $\Delta S_{\text{univ}} = S_2 - S_1 - \dfrac{Q_R}{T_0} = 0$

$$\therefore \quad Q_R = T_0 (S_2 - S_1) \tag{8.13}$$

For an irreversible process,

$$\Delta S_{\text{univ}} > 0$$

$$\therefore \quad S_2 - S_1 - \frac{Q_I}{T_0} > 0$$

$$\therefore \quad Q_I < T_0 (S_2 - S_1) \tag{8.14}$$

From Eqs (8.13) and (8.14), $$Q_R > Q_I \tag{8.15}$$

Therefore, from Eqs (8.12) and (8.15), $$W_R > W_I \tag{8.16}$$

Therefore, the work done by a closed system by interacting only with the surroundings at p_0, T_0 in a reversible process is always more than that done by it in an irreversible process between the same end states.

8.4.1 Work Done in all Reversible Processes is the Same

Let us assume two reversible processes R_1 and R_2 between the same end states 1 and 2 undergone by a closed system by exchanging energy only with the surroundings (Fig. 8.10). Let one of the processes be reversed.

Then the system would execute a cycle 1–2–1 and produce network represented by the area enclosed by exchanging energy with only one reservoir, i.e. the surroundings. This violates the Kelvin-Planck statement. Therefore, *the two reversible processes must coincide and produce equal amounts of work.*

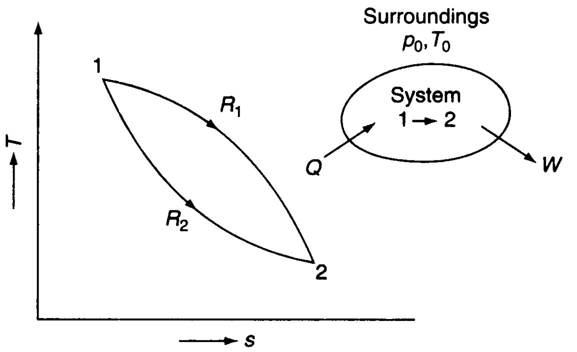


Fig. 8.10 *Equal work done in all reversible processes between the same end states*

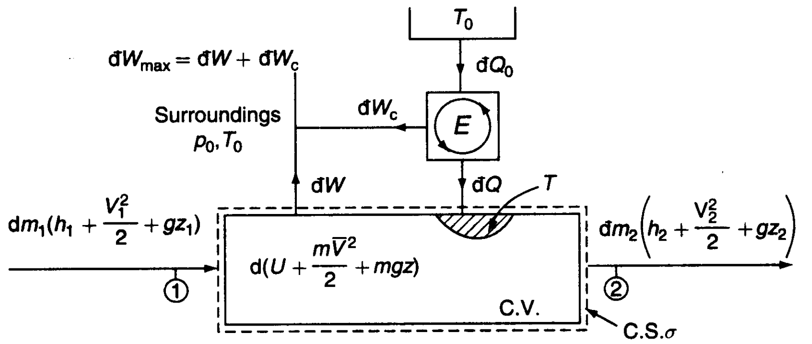
8.5 REVERSIBLE WORK BY AN OPEN SYSTEM EXCHANGING HEAT ONLY WITH THE SURROUNDINGS

Let us consider an open system exchanging energy only with the surroundings at constant temperature T_0 and at constant pressure p_0 (Fig. 8.11). A mass dm_1 enters the system at state 1, a mass dm_2 leaves the system at state 2, an amount of heat $đQ$ is absorbed by the system, an amount of work $đW$ is delivered by the system, and the energy of the system (control volume) changes by an amount $d(E)_\sigma$. Applying the first law, we have

$$đQ + dm_1\left(h_1 + \frac{V_1^2}{2} + gz_1\right) - dm_2\left(h_2 + \frac{V_2^2}{2} + gz_2\right) - đW$$

$$= dE_\sigma = d\left[U + \frac{mV^2}{2} + mgz\right]_\sigma \tag{8.17}$$

For maximum work, the process must be entirely reversible. There is a temperature difference between the control volume and the surroundings. To make the heat transfer process reversible, let us assume a reversible heat engine E operating between the two. Again, the temperature of the fluid in the control volume may be different at different points. It is assumed that heat transfer occurs at points of the control surface σ where the temperature is T. Thus in an infinitesimal reversible process an amount of heat $đQ_0$ is absorbed by the engine E



Reversible work done by an open system while exchanging heat only with the surroundings

from the surroundings at temperature T_0, an amount of heat $\text{đ}Q$ is rejected by the engine reversibly to the system where the temperature is T, and an amount of work $\text{đ}W_c$ is done by the engine. For a reversible engine,

$$\frac{\text{đ}Q_0}{T_0} = \frac{\text{đ}Q}{T}$$

$$\therefore \quad \text{đ}W_c = \text{đ}Q_0 - \text{đ}Q = \text{đ}Q \cdot \frac{T_0}{T} - \text{đ}Q$$

or

$$\text{đ}W_c = \text{đ}Q\left(\frac{T_0}{T} - 1\right) \tag{8.18}$$

The work $\text{đ}W_c$ is always positive and is independent of the direction of heat flow. When $T_0 > T$, heat will flow from the surroundings to the system, $\text{đ}Q$ is positive and hence $\text{đ}W_c$ in Eq. (8.18) would be positive. Again, when $T_0 < T$, heat will flow from the system to the surroundings, $\text{đ}Q$ is negative, and hence $\text{đ}W_c$ would be positive.

Now, since the process is reversible, the entropy change of the system will be equal to the net entropy transfer, and $S_{gen} = 0$. Therefore,

$$\underset{\text{Entropy change}}{dS} = \underset{\text{Entropy transfer with heat}}{\frac{\text{đ}Q}{T}} + \underset{\text{Entropy transfer with mass}}{dm_1 s_1 - dm_2 s_2}$$

$$\therefore \quad \frac{\text{đ}Q}{T} = dS - dm_1 s_1 + dm_1 s_2 \tag{8.19}$$

Now, the maximum work is equal to the sum of the system work dW and the work dW_c of the reversible engine E, $\text{đ}W_{max} = \text{đ}W_{rev} = \text{đ}W + \text{đ}W_c$ (8.20)

From Eq. (8.18), $\text{đ}W_{max} = dW + dQ\left(\frac{T_0}{T} - 1\right)$ (8.21)

Substituting Eq. (8.17) for $\text{đ}W$ in Eq. (8.21),

$$\text{đ}W_{max} = \text{đ}Q + dm_1\left(h_1 + \frac{V_1^2}{2} + gz_1\right) - dm_2\left(h_2 + \frac{V_2^2}{2} + gz_2\right) - d\left[U + \frac{mV^2}{2} + mgz\right]_\sigma + \text{đ}Q\left(\frac{T_0}{T} - 1\right)$$

$$= dm_1\left(h_1 + \frac{V_1^2}{2} + gz_1\right) - dm_2\left(h_2 + \frac{V_2^2}{2} + gz_2\right) - d\left[U + \frac{mV^2}{2} + mgz\right]_\sigma + \frac{\text{đ}Q}{T}T_0 \tag{8.22}$$

On substituting the value of $\text{đ}Q/T$ from Eq. (8.19),

$$\text{đ}W_{\max} = \mathrm{d}m_1\left(h_1 + \frac{V_1^2}{2} + gz_1\right) - \mathrm{d}m_2\left(h_2 + \frac{V_2^2}{2} + gz_2\right) - \mathrm{d}\left[U + \frac{mV^2}{2} + mgz\right]_\sigma + T_0\,(\mathrm{d}S - \mathrm{d}m_1 s_1 + \mathrm{d}m_2 s_2)$$

$$\therefore\ \text{đ}W_{\max} = \mathrm{d}m_1\left(h_1 - T_0 s_1 + \frac{V_1^2}{2} + gz_1\right) - \mathrm{d}m_2\left(h_2 - T_0 s_2 + \frac{V_2^2}{2} + gz_2\right) - \mathrm{d}\left[U - T_0 S + \frac{mV^2}{2} + mgz\right]_\sigma \quad (8.23)$$

Equation (8.23) is the general expression for the maximum work of an open system which exchanges heat only with the surroundings at T_0, p_0.

8.5.1 Reversible Work in a Steady Flow Process

For a steady flow process $\mathrm{d}m_1 = \mathrm{d}m_2 = \mathrm{d}m$

and
$$\mathrm{d}\left[U - T_0 S + \frac{mV^2}{2} + mgz\right]_\sigma = 0$$

Equation (8.24) reduces to

$$\text{đ}W_{\max} = \mathrm{d}m\left[\left(h_1 - T_0 s_1 + \frac{V_1^2}{2} + gz_1\right) - \left(h_2 - T_0 s_2 + \frac{V_2^2}{2} + gz_2\right)\right] \quad (8.24)$$

For total mass flow, the integral form of Eq. (8.24) becomes

$$W_{\max} = \left(H_1 - T_0 S_1 + \frac{mV_1^2}{2} + mgz_1\right) - \left(H_2 - T_0 S_2 + \frac{mV_2^2}{2} + mgz_2\right) \quad (8.25)$$

The expression $(H - T_0 S)$ is called the *Keenan function, B*.

$$\therefore\quad W_{\max} = \left(B_1 + \frac{mV_1^2}{2} + mgz_1\right) - \left(B_2 + \frac{mV_2^2}{2} + mgz_2\right)$$
$$= \Psi_1 - \Psi_2 \quad (8.26)$$

where Ψ is called the availability function of a steady flow process given by $\Psi = B + \dfrac{mV^2}{2} + mgz$

On a unit mass basis,

$$W_{\max} = \left(h_1 - T_0 s_1 + \frac{V_1^2}{2} + gz_1\right) - \left(h_1 - T_0 s_2 + \frac{V_2^2}{2} + gz_2\right) = \left(b_1 + \frac{V_1^2}{2} + gz_1\right) - \left(b_2 + \frac{V_2^2}{2} + gz_2\right) \quad (8.27)$$

If K.E. and P.E. changes are neglected, Eqs (8.26) and (8.27) reduce to

$$W_{\max} = B_1 - B_2 = (H_1 - T_0 S_1) - (H_2 - T_0 S_2) = (H_1 - H_2) - T_0(S_1 - S_2) \quad (8.28)$$

and per unit mass

$$W_{\max} = b_1 - b_2 = (h_1 - h_2) - T_0\,(s_1 - s_2) \quad (8.29)$$

8.5.2 Reversible Work in a Closed System

For a *closed system*, $\mathrm{d}m_1 = \mathrm{d}m_2 = 0$

Equation (8.23) then becomes $\mathrm{d}W_{\max} = -\,\mathrm{d}\left[U - T_0 S + \dfrac{mV^2}{2} + mgz\right]_\sigma = -\,\mathrm{d}(E - T_0 S)_\sigma$

where $$E = U + \frac{mV^2}{2} + mgz$$

For a change of state of the system from the initial state 1 to the final state 2,

$$\begin{aligned} W_{\max} &= E_1 - E_2 - T_0(S_1 - S_2) \\ &= (E_1 - T_0S_1) - (E_2 - T_0S_2) \end{aligned} \tag{8.30}$$

If the K.E. and P.E. changes are neglected, Eq. (8.30) reduces to

$$W_{\max} = (U_1 - T_0S_1) - (U_2 - T_0S_2) \tag{8.31}$$

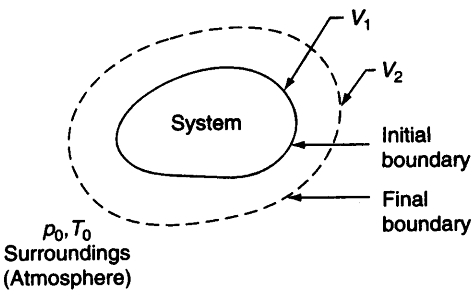
For unit mass of fluid,

$$\begin{aligned} W_{\max} &= (u_1 - u_2) - T_0(s_1 - s_2) \\ &= (u_1 - T_0s_1) - (u_2 - T_0s_2) \end{aligned} \tag{8.32}$$

8.6 USEFUL WORK

All of the work W of the system with a flexible boundary would not be available for delivery, since a certain portion of it would be spent in pushing out the atmosphere (Fig. 8.12). The useful work is defined as the actual work delivered by a system less the work performed on the atmosphere. If V_1 and V_2 are the initial and final volume of the system and p_0 is the atmospheric pressure, then the work done on the atmosphere is $p_0(V_2 - V_1)$. Therefore, the useful work W_u becomes

$$W_u = W_{act} - p_0(V_2 - V_1) \tag{8.33}$$



Work done by a closed system in pushing out the atmosphere

Similarly, the *maximum useful work* will be

$$(W_u)_{\max} = W_{\max} - p_0(V_2 - V_1) \tag{8.34}$$

In differential form

$$(dW_u)_{\max} = dW_{\max} - p_0\,dV \tag{8.35}$$

In a steady flow system, the volume of the system does not change. Hence, the maximum useful work would remain the same, i.e., *no work is done on the atmosphere*, or

$$(\text{đ}W_u)_{\max} = \text{d}W_{\max} \tag{8.36}$$

But in the case of an unsteady-flow open system or a closed system, the volume of the system changes. Hence, when a system exchanges heat only with the atmosphere, the maximum useful work becomes

$$(\text{đ}W_u)_{\max} = \text{đ}W_{\max} - p_0\,\text{d}V$$

Substituting $dW_{\max}$ from Eq. (8.23),

$$\begin{aligned} (\text{đ}W_u)_{\max} &= \text{d}m_1\left(h_1 - T_0s_1 + \frac{V_1^2}{2} + gz_1\right) - \text{d}m_2\left(h_2 - T_0s_2 + \frac{V_2^2}{2} + gz_2\right) \\ &\quad - \text{d}\left[U + p_0V - T_0S + \frac{mV^2}{2} + mgz\right]_\sigma \end{aligned} \tag{8.37}$$

This is the maximum useful work for an *unsteady open system*. For the closed system, Eq. (8.37) reduces to

$$(\text{d}W_u)_{\max} = -\text{d}\left[U + p_0V - T_0S + \frac{mV^2}{2} + mgz\right]_\sigma = -\,\text{d}[E + p_0V - T_0S]_\sigma \tag{8.38}$$

$$\therefore \qquad (W_u)_{max} = E_1 - E_2 + p_0(V_1 - V_2) - T_0(S_1 - S_2) \tag{8.39}$$

If K.E. and P.E. changes are neglected, Eq. (8.39) becomes

$$(W_u)_{max} = U_1 - U_2 + p_0(V_1 - V_2) - T_0(S_1 - S_2) \tag{8.40}$$

This can also be written in the following form

$$(W_u)_{max} = (U_1 + p_0V_1 - T_0S_1) - (U_2 + p_0V_2 - T_0S_2) = \phi_1 - \phi_2 \tag{8.41}$$

where ϕ is called the *availability function for a closed system* given by

$$\phi = U + p_0V - T_0S$$

The useful work per unit mass becomes

$$(W_u)_{max} = (u_1 + p_0v_1 - T_0s_1) - (u_2 + p_0v_2 - T_0s_2) \tag{8.42}$$

8.6.1 Maximum Useful Work Obtainable when the System Exchanges Heat with a Thermal Reservoir in Addition to the Atmosphere

If the open system discussed in Sec. 8.5 exchanges heat with a thermal energy reservoir at temperature T_R in addition to the atmosphere, the *maximum useful work* will be increased by $\text{đ}Q_R\left(1 - \frac{T_0}{T_R}\right)$, where $\text{đ}Q_R$ is the heat received by the system. For a steady flow process,

$$(W_u)_{max} = W_{max} = \left(H_1 - T_0S_1 + \frac{m\text{V}_1^2}{2} + mgz_1\right) - \left(H_2 - T_0S_2 + \frac{m\text{V}_2^2}{2} + mgz_2\right) + Q_R\left(1 - \frac{T_0}{T_R}\right)$$

$$= \psi_1 - \psi_2 + Q_R\left(1 - \frac{T_0}{T_R}\right) \tag{8.43}$$

For a closed system

$$(W_u)_{max} = W_{max} - p_0(V_2 - V_1) + Q_R\left(1 - \frac{T_0}{T_R}\right)$$

or

$$(W_u)_{max} = E_1 - E_2 + p_0(V_1 - V_2) - T_0(S_1 - S_2) + Q_R\left(1 - \frac{T_0}{T_R}\right) \tag{8.44}$$

If K.E. and P.E. changes are neglected, then for a *steady flow process*:

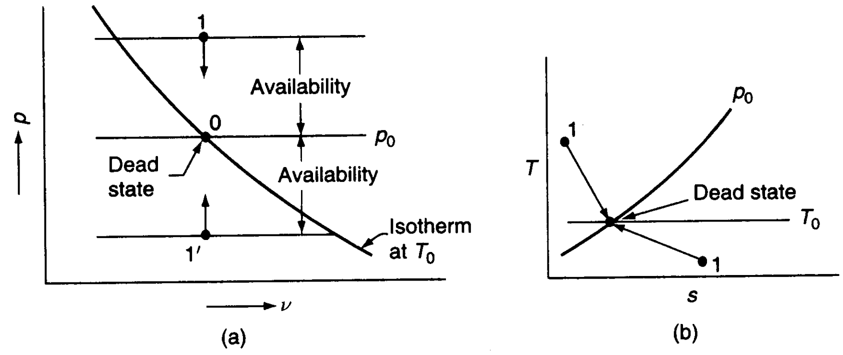
$$(W_u)_{max} = (H_1 - H_2) - T_0(S_1 - S_2) + Q_R\left(1 - \frac{T_0}{T_R}\right) \tag{8.45}$$

and for a *closed system:*

$$(W_u)_{max} = U_1 + U_2 - p_0(V_1 - V_2) - T_0(S_1 - S_2) + Q_R\left(1 - \frac{T_0}{T_R}\right) \tag{8.46}$$

8.7 DEAD STATE

If the state of a system departs from that of the surroundings, an opportunity exists for producing work (Fig. 8.13). However, as the system changes its state towards that of the surroundings, this opportunity diminishes, and it ceases to exist when the two are in equilibrium with each other. When the system is in equilibrium with the surroundings, it must be in pressure and temperature equilibrium with the surroundings, i.e., at p_0 and T_0. It must also be in chemical equilibrium with the surroundings, i.e., there should not be any chemical reaction or mass transfer. The system must have zero velocity and minimum potential energy. This state of the system is known as the *dead state*, which is designated by affixing subscript '0' to the properties. Any change in the state



Available work of a system decreases as its state approaches P_0, T_0

of the system from the dead state is a measure of the available work that can be extracted from it. Farther the initial point of the system from the dead state in terms of *p*, *t* either above or below it, higher will be the available energy or exergy of the system (Fig. 8.13). All spontaneous processes terminate at the dead state.

8.8 AVAILABILITY

Whenever useful work is obtained during a process in which a finite system undergoes a change of state, the process must terminate when the pressure and temperature of the system have become equal to the pressure and temperature of the surroundings, p_0 and T_0, i.e., when the system has reached the dead state. An air engine operating with compressed air taken from a cylinder will continue to deliver work till the pressure of air in the cylinder becomes equal to that of the surroundings, p_0. A certain quantity of exhaust gases from an internal combustion engine used as the high temperature source of a heat engine will deliver work until the temperature of the gas becomes equal to that of the surroundings, T_0.

The availability (A) of a given system is defined as *the maximum useful work (total work minus pdV work) that is obtainable in a process in which the system comes to equilibrium with its surroundings.* Availability is thus a composite property depending on the state of both the system and surroundings.

8.8.1 Availability in a Steady Flow Process

The reversible (maximum) work associated with a steady flow process for a single flow is given by Eq. (8.25)

$$W_{\text{rev}} = \left(H_1 - T_0S_1 + \frac{m\text{V}_1^2}{2} + mgz_1\right) - \left(H_2 - T_0S_2 + \frac{m\text{V}_2^2}{2} + mgz_2\right)$$

With a given state for the mass entering the control volume, the maximum useful work obtainable (i.e., the availability) would be when this mass leaves the control volume in equilibrium with the surroundings (i.e., at the dead state). Since there is no change in volume, no work will be done on the atmosphere. Let us designate the initial state of the mass entering the C.V. with parameters having *no subscript* and the final dead state of the mass leaving the C.V. with parameters having subscript 0. The maximum work or availability, *A*, would be

$$A = \left(H - T_0S + \frac{m\text{V}^2}{2} + mgz\right) - (H_0 - T_0S_0 + mgz_0) = \psi - \psi_0 \tag{8.47}$$

where ψ is called the *availability function for a steady flow system* and $\text{V}_0 = 0$. This is the availability of a system at any state as it enters a C.V. in a steady flow process. The availability per unit mass would be

$$a = \left(h - T_0 s + \frac{\mathrm{V}^2}{2} + gz\right) - (h_0 - T_0 s_0 + gz) = \psi - \psi_0 \tag{8.48}$$

If subscripts 1 and 2 denote the states of a system entering and leaving a C.V., the decrease in availability or maximum work obtainable for the given system-surroundings combination would be

$$W_{\max} = a_1 - a_2 = \psi_1 - \psi_2 = \left[\left(h_1 - T_0 s_1 + \frac{\mathrm{V}_1^2}{2} + gz_1\right) - \left(h_0 - T_0 s_0 + gz_0\right)\right]$$

$$- \left[\left(h_2 - T_0 s_0 + \frac{\mathrm{V}_2^2}{2} + gz_2\right) - \left(h_0 - T_0 s_0 + gz_0\right)\right] = (h_1 - h_2) - T_0\,(s_1 - s_2) + \frac{\mathrm{V}_1^2 - \mathrm{V}_2^2}{2} + g(z_1 - z_2) \tag{8.49}$$

If K.E. and P.E. changes are neglected,

$$W_{\max} = (h_1 - T_0 s_1) - (h_2 - T_0 s_2) = b_1 - b_2$$

where *b* is the specific Keenan function.

If more than one flow into and out of the C.V. is involved.

$$W_{\max} = \sum_{\mathrm{i}} m_{\mathrm{i}}\psi_{\mathrm{i}} - \sum_{\mathrm{i}} m_{\mathrm{e}}\psi_{\mathrm{e}}$$

8.8.2 Availability in a Nonflow Process

Let us consider a closed system and denote its initial state by parameters without any subscript and the final dead state with subscript '0'. The availability of the system *A*, i.e., the maximum useful work obtainable as the system reaches the dead state, is given by Eq. (8.39).

$$A = (W_{\mathrm{u}})_{\max} = E - E_0 + p_0(V - V_0) - T_0(S - S_0)$$

$$= \left(U + \frac{m\mathrm{V}^2}{2} + mgz\right) - (U_0 + mgz_0) + p_0(V - V_0) - T_0(S - S_0) \tag{8.50}$$

If K.E. and P.E. changes are neglected and for unit mass, the availability becomes

$$a = u - u_0 + p_0(v - v_0) - T_0(s - s_0) = (u + p_0 v - T_0 s) - (u_0 + p_0 v_0 - T_0 s_0) = \phi - \phi_0 \tag{8.51}$$

where ϕ is the availability function of the closed system.

If the system undergoes a change of state from 1 to 2, the decrease in availability will be

$$a = (\phi_1 - \phi_0) - (\phi_2 - \phi_0) = \phi_1 - \phi_2 = (u_1 - u_2) + p_0(v_1 - v_2) - T_0(s_1 - s_2) \tag{8.52}$$

This is the maximum useful work obtainable under the given surroundings.

8.9 AVAILABILITY IN CHEMICAL REACTIONS

In many chemical reactions the reactants are often in pressure and temperature equilibrium with the surroundings (before the reaction takes place) and so are the products after the reaction. An internal combustion engine can be cited as an example of such a process if we visualize the products being cooled to atmospheric temperature T_0 before being discharged from the engine.

(a) Let us first consider a system which is in temperature equilibrium with the surroundings before and after the process. The maximum work obtainable during a change of state is given by Eq. (8.30),

$$W_{\max} = E_1 - E_2 - T_0(S_1 - S_2) = \left(U_1 + \frac{m\mathrm{V}_1^2}{2} + mgz_1\right) - \left(U_2 + \frac{m\mathrm{V}_2^2}{2} + mgz_2\right) - T_0(S_1 - S_2)$$

If K.E. and P.E. changes are neglected,

$$W_{\max} = U_1 - U_2 - T_0(S_1 - S_2)$$

Since the initial and final temperatures of the system are the same as that of the surroundings, $T_1 = T_2 = T_0 = T$, say, then

$$(W_T)_{\max} = (U_1 - U_2)_T - T(S_1 - S_2)_T \tag{8.53}$$

Let a property called *Helmholtz function F* be defined by the relation

$$F = U - TS \tag{8.54}$$

Then for two equilibrium states 1 and 2 at the same temperature T,

$$(F_1 - F_2)_T = (U_1 - U_2)_T - T(S_1 - S_2)_T \tag{8.55}$$

From Eqs (8.53) and (8.55),

$$(W_T)_{\max} = (F_1 - F_2)_T \tag{8.56}$$

or

$$W_T \leq (F_1 - F_2)_T \tag{8.57}$$

The work done by a system in any process between two equilibrium states at the same temperature during which the system exchanges heat only with the environment is equal to or less than the decrease in the Helmholtz function of the system during the process. The maximum work is done when the process is reversible and the equality sign holds. If the process is irreversible, the work is less than the maximum.

(b) Let us now consider a system which is in both pressure and temperature equilibrium with the surroundings before and after the process. When the volume of the system increases some work is done by the system against the surroundings ($p\mathrm{d}V$ work), and this is not available for doing useful work. The availability of the system, as defined by Eq. (8.50), neglecting the K.E. and P.E. changes, can be expressed in the form

$$A = (W_u)_{\max} = (U + p_0V - T_0S) - (U_0 + p_0V_0 - T_0S_0) = \phi - \phi_0$$

The maximum work obtainable during a change of state is the decrease in availability of the system, as given by Eq. (8.52) for unit mass.

$$\therefore \quad (W_u)_{\max} = A_1 - A_2 = \phi_1 - \phi_2 = (U_1 - U_2) + p_0(V_1 - V_2) - T_0(S_1 - S_2)$$

If the initial and final equilibrium states of the system are at the same pressure and temperature of the surroundings, say $p_1 = p_2 = p_0 = p$, and $T_1 = T_2 = T_0 = T$. Then,

$$(W_u)_{\max} = (U_1 - U_2)_{p,T} + p(V_1 - V_2)_{p,T} - T(S_1 - S_2)_{p,T} \tag{8.58}$$

The *Gibbs function G* is defined as

$$G = H - TS = U + pV - TS \tag{8.59}$$

Then for two equilibrium states at the same pressure p and temperature T

$$(G_1 - G_2)_{p,T} = (U_1 - U_2)_{p,T} + p(V_1 - V_2)_{p,T} - T(S_1 - S_2)_{p,T} \tag{8.60}$$

From Eqs (8.58) and (8.60),

$$\underset{p,T}{(W_u)_{\max}} = (G_1 - G_2)_{p,T} \tag{8.61}$$

$$\therefore \quad (W_u)_{p,T} \leq (G_1 - G_2)_{p,T} \tag{8.62}$$

The decrease in the Gibbs function of a system sets an upper limit to the work that can be performed, exclusive of $p\mathrm{d}V$ work, in any process between two equilibrium states at the same temperature and pressure, provided the system exchanges heat only with the environment which is at the same temperature and pressure as the end states of the system. If the process is irreversible, the useful work is less than the maximum.

8.10 IRREVERSIBILITY AND GOUY-STODOLA THEOREM

The actual work done by a system is always less than the idealized reversible work, and the difference between the two is called the irreversibility of the process.

$$I = W_{\max} - W \tag{8.63}$$

This is also sometimes referred to as '*degradation*' or '*dissipation*'.

For a non-flow process between the equilibrium states, when the system exchanges heat only with the environment

$$I = [(U_1 - U_2) - T_0(S_1 - S_2)] - [(U_1 - U_2) + Q] = T_0(S_2 - S_1) - Q = T_0(\Delta S)_{\text{system}} + T_0(\Delta S)_{\text{surr}}$$

$$= T_0[(\Delta S)_{\text{system}} + (\Delta S)_{\text{surr}}] \qquad (8.64)$$

$$\therefore \qquad I \geq 0$$

Similarly, for the steady flow process

$$I = W_{\max} - W = \left[\left(B_1 + \frac{mV_1^2}{2} + mgZ_1\right) - \left(B_2 + \frac{mV^2}{2} + mgZ_2\right)\right]$$

$$- \left[\left(H_1 + \frac{mV_1^2}{2} + mgZ_1\right) - \left(H_2 + \frac{mV_2^2}{2} + mgZ_2\right) + Q\right] = T_0(S_2 - S_1) - Q = T_0(\Delta S)_{\text{system}} + T_0(\Delta S)_{\text{surr}}$$

$$= T_0(\Delta S_{\text{system}} + S_{\text{surr}}) = T_0 \Delta S_{\text{univ}} \qquad (8.65)$$

The same expression for irreversibility applies to both flow and non-flow processes. The quantity $T_0(\Delta S_{\text{system}} + \Delta S_{\text{surr}})$ represents an increase in unavailable energy (or anergy).

The *Gouy-Stodola theorem* states that the rate of loss of available energy or exergy in a process is proportional to the rate of entropy generation, S_{gen}. If Eqs (8.64) and (8.65) are written in the rate form,

$$\dot{I} = \dot{W}_{\text{lost}} = T_0 \Delta \dot{S}_{\text{univ}} = T_0 \dot{S}_{\text{gen}} \qquad (8.66)$$

This is known as the Gouy-Stodola equation. A thermodynamically efficient process would involve minimum exergy loss with minimum rate of entropy generation.

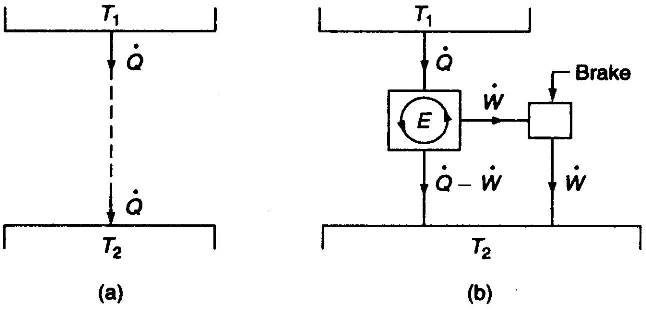
8.10.1 Applications of Gouy-Stodola Equation

(a) Heat Transfer through a Finite Temperature Difference If heat transfer $\dot{Q}$ occurs from the hot reservoir at temperature T_1 to the cold reservoir at temperature T_2 (Fig. 8.14a)

$$S_{\text{gen}} = \frac{\dot{Q}}{T_2} - \frac{\dot{Q}}{T_1} = \dot{Q}\,\frac{T_1 - T_2}{T_1 T_2} \quad \text{and} \quad \dot{W}_{\text{lost}} = \dot{Q}\left(1 - \frac{T_2}{T_1}\right) = \dot{Q}\,\frac{T_1 - T_2}{T_1}$$

$$\therefore \qquad \dot{W}_{\text{lost}} = T_2\, S_{\text{gen}}$$

If the heat transfer $\dot{Q}$ from T_1 to T_2 takes place through a reversible engine E, the entire work output is $\dot{W}$ dissipated in the brake, from which an equal amount of heat is rejected to the reservoir at T_2 (Fig. 8.14b). Heat transfer through a finite temperature difference is equivalent to the destruction of its availability.



(a) (b)

Destruction of available work or exergy by heat transfer through a finite temperature difference

(b) Flow with Friction Let us consider the steady and adiabatic flow of an *ideal gas* through the segment of a pipe (Fig. 8.15a).

By the first law,

$$h_1 = h_2$$

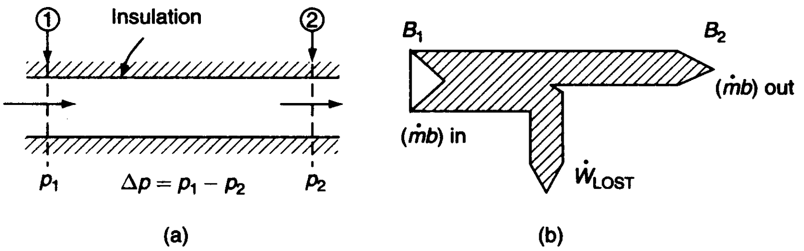
and by the second law,

$$T\mathrm{d}s = \mathrm{d}h - v\mathrm{d}p$$

$$\int_1^2 \mathrm{d}s = \int_1^2 \frac{\mathrm{d}h}{T} - \int_1^2 \frac{v}{T}\mathrm{d}p = -\int_1^2 \frac{v}{T}\mathrm{d}p$$

$$\therefore \quad \dot{S}_{\text{gen}} = \int_1^2 \dot{m}\mathrm{d}s = -\int_{p1}^{p2} \dot{m}R\frac{\mathrm{d}p}{p} = -\dot{m}R\ln\frac{p_2}{p_1} = -\dot{m}R\ \ln\left(1-\frac{\Delta p}{p_1}\right) = -\dot{m}R\left(-\frac{\Delta p}{p_1}\right)$$

$$= \dot{m}R\frac{\Delta p}{p_1} \tag{8.67}$$



Irreversibility in a duct due to fluid friction

where $\quad \ln\left(1-\dfrac{\Delta p}{p_1}\right) \simeq -\dfrac{\Delta p}{p_1}$, since $\dfrac{\Delta p}{p_1} < 1$

and higher terms are neglected.

$$\dot{W}_{\text{lost}} = \dot{B}_1 - \dot{B}_2 = \dot{m}\left[(h_1 - T_0 s_1) - (h_2 - T_0 s_2)\right] = \dot{m}T_0(s_2 - s_1) = T_0\,\dot{S}_{\text{gen}} = \dot{m}RT_0\frac{\Delta p}{p_1} \tag{8.68}$$

The decrease in availability or lost work is proportional to the pressure drop (Δp) and the mass flow rate ($\dot{m}$). It is shown on the right (Fig. 8.15b) by the *Grassmann diagram*, the width being proportional to the availability (or exergy) of the stream. It is an adaptation of the *Sankey diagram* used for energy transfer in a plant.

(c) Mixing of Two Fluids Two streams 1 and 2 of an incompressible fluid or an ideal gas mix adiabatically at constant pressure (Fig. 8.16).

Here, $\quad \dot{m}_1 + \dot{m}_2 = \dot{m}_3 = \dot{m}$ (say)

Let $\quad x = \dfrac{\dot{m}_1}{\dot{m}_1 + \dot{m}_2}$

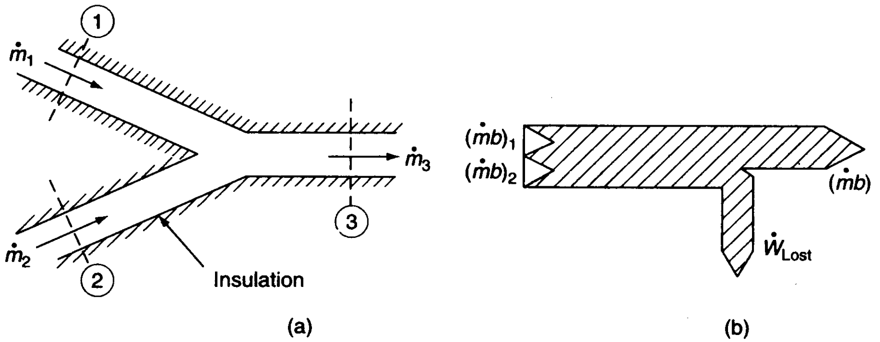
By the first law,

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2)h_3 \quad \text{or} \quad xh_1 + (1-x)h_2 = h_3$$

The preceding equation may be written in the following form, since enthalpy is a function of temperature.

$$xT_1 + (1-x)T_2 = T_3$$

$$\therefore \quad \frac{T_3}{T_1} = x + (1-x)\tau \tag{8.69}$$



(a) (b)

Irreversibility due to mixing

where $$\tau = \frac{T_2}{T_1}$$

By the second law,

$$\dot S_{gen} = \dot m_3\, s_3 - \dot m_1\, s_1 - \dot m_2\, s_2 = \dot m s_3 - x\dot m s_1 - (1-x)\,\dot m s_2$$

∴ $$\frac{\dot S_{gen}}{\dot m} = (s_3 - s_2) + x(s_2 - s_1) = c_p \ln\frac{T_3}{T_2} + xc_p \ln\frac{T_2}{T_1}$$

or $$\frac{\dot S_{gen}}{\dot m c_p} = \ln\left(\frac{T_3}{T_2}\right)\left(\frac{T_2}{T_1}\right)^x$$

or $$N_s = \ln\left(\frac{T_3}{T_1}\right)\frac{T_1^{1-x}}{T_2^{1-x}}$$

or $$N_s = \ln\frac{T_3/T_1}{(T_2/T_1)^{1-x}} \tag{8.70}$$

where N_s is a dimensionless quantity, called the *entropy generation number*, given by $\dot S_{gen}/\dot m c_p$.

Substituting T_3/T_1 from Eq. (8.69) in Eq. (8.70),

$$N_s = \ln\frac{x + \tau(1-x)}{\tau^{-1-x}} \tag{8.71}$$

If $x = 1$ or $\tau = 1$, N_s becomes zero in each case. The magnitude of N_s depends on x and τ. The rate of loss of exergy due to mixing would be

$$\dot W_{lost} = \dot I = T_0\, \dot m c_p \ln\frac{x + \tau(1-x)}{\tau^{1-x}} \tag{8.72}$$

8.11 AVAILABILITY OR EXERGY BALANCE

The availability or exergy is the maximum useful work obtainable from a system as it reaches the dead state (p_0, t_0). Conversely, availability or exergy can be regarded as the minimum work required to bring the closed system from the dead state to the given state. The value of exergy cannot be negative. If a closed system were at any state other than the dead state, the system would be able to change its state spontaneously toward the dead state. This tendency would stop when the dead state is reached. No work is done to effect such a spontaneous change. Since any change in state of the closed system to the dead state can be accomplished with zero work, the *maximum work* (*or exergy*) *cannot be negative.*

While energy is always conserved, exergy is not generally conserved, but is destroyed by irreversibilities. When the closed system is allowed to undergo a spontaneous change from the given state to the dead state, its exergy is completely destroyed without producing any useful work. The potential to develop work that exists originally at the given state is thus completely wasted in such a spontaneous process. Therefore, at steady state:

1. Energy in – Energy out = 0
2. Exergy in – Exergy out = Exergy destroyed

8.11.1 Exergy Balance for a Closed System

For a closed system, availability or exergy transfer occurs through heat and work interactions (Fig. 8.17).

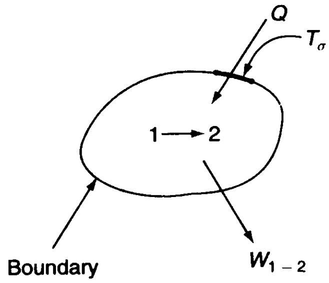


Fig. 8.17 *Exergy balance for a closed system*

1st law:
$$E_2 - E_1 = \int_1^2 đQ - W_{1-2} \tag{8.73}$$

2nd law:
$$S_2 - S_1 - \int_1^2 \left[\frac{đQ}{T}\right] = S_{\text{gen}}$$

or
$$T_0(S_2 - S_1) - T_0\int_1^2 \left[\frac{đQ}{T}\right]_\sigma = T_0\, S_{\text{gen}} \tag{8.74}$$

Subtracting Eq. (8.74) from Eq. (8.73),

$$E_2 - E_1 - T_0(S_2 - S_1) = \int_1^2 đQ - W_{1-2} - T_0\int_1^2 \left[\frac{đQ}{T}\right] - T_0 S_{\text{gen}}$$

$$= \int_1^2 \left[1 - \frac{T_0}{T_\sigma}\right] đQ - W_{1-2} - T_0 S_{\text{gen}}$$

Since,

$$A_2 - A_1 = E_2 - E_1 + p_0(V_2 - V_1) - T_0(S_2 - S_1)$$

$$\underbrace{A_2 - A_1}_{\text{Change in exergy}} = \underbrace{\int_1^2 \left[1 - \frac{T_0}{T_\sigma}\right] đQ}_{\text{Exergy transfer with heat}} - \underbrace{[W_{1-2} - p_0(V_2 - V_1)]}_{\text{Exergy transfer with work}} - \underbrace{T_0 S_{\text{gen}}}_{\text{Exergy destruction}} \tag{8.75}$$

In the form of the rate equation,

$$\underbrace{\frac{dA}{d\tau}}_{\substack{\text{Rate of}\\ \text{change of}\\ \text{exergy}}} = \underbrace{\sum_j \left[1 - \frac{T_0}{T_j}\right] \dot{Q}_j}_{\substack{\text{Rate of exergy}\\ \text{transfer with heat}\\ Q_j \text{ at the boundary}\\ \text{where the instantaneous}\\ \text{temperature is } T_j}} - \underbrace{\left[W - p_0 \frac{dV}{d\tau}\right]}_{\substack{\text{Rate of exergy}\\ \text{transfer as work}\\ \text{where } dV/dt \text{ is}\\ \text{the rate of}\\ \text{change of system}\\ \text{volume}}} - \underbrace{\dot{I}}_{\substack{\text{Rate of exergy}\\ \text{loss due to}\\ \text{irreversibilities}\\ (= T_0 \dot{S}_{\text{gen}})}} \tag{8.76}$$

For an isolated system, the exergy balance, Eq. (8.76), gives

$$\Delta A = -I \tag{8.77}$$

Since $I > 0$, the only processes allowed by the second law are those for which the exergy of the isolated system decreases. In other words,

The exergy of an isolated system can never increase.

It is the *counterpart of the entropy principle* which states that the entropy of an isolated system can never decrease.

The exergy balance of a system can be used to determine the locations, types and magnitudes of losses (waste) of the potential of energy resources (fuels) and find ways and means to reduce these losses for making the system more energy efficient and for more effective use of fuel.

8.11.2 Exergy Balance for a Steady Flow System

1st law:

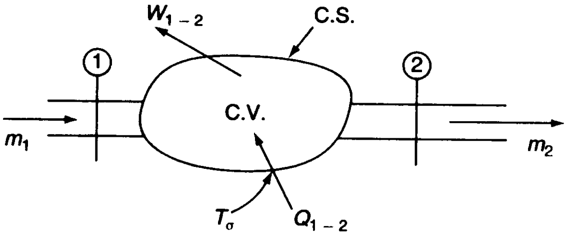
$$H_1 + \frac{mV_1^2}{2} + mgZ_1 + Q_{1-2} = H_2 + \frac{mV_2^2}{2} + mgZ_2 + W_{1-2} \qquad (8.78)$$

2nd law:

$$S_1 + \int_1^2 \left[\frac{\text{đ}Q}{T}\right] - S_2 = S_{\text{gen}}$$

or

$$T_0(S_1 - S_2) + T_0 \int_1^2 \left[\frac{\text{đ}Q}{T}\right]_\sigma = T_0 S_{\text{gen}} = I \qquad (8.79)$$



[redacted] *Exergy balance for a steady flow system*

From Eqs (8.78) and (8.79),

$$H_2 - H_1 - T_0(S_2 - S_1) + \frac{m\,V_2^2 - V_1^2}{2} + mg(Z_2 - Z_1) = \int_1^2 \left(1 - \frac{T_0}{T_\sigma}\right) \text{đ}Q \;- W_{1-2} - I \qquad (8.80)$$

or

$$A_2 - A_1 = \int_1^2 \left(1 - \frac{T_0}{T_\sigma}\right) \text{đ}Q - W_{1-2} - I \qquad (8.81)$$

In the form of rate equation at steady state:

$$\sum_j \left(1 - \frac{T_0}{T_j}\right) \dot{Q}_j - \dot{W}_{\text{C.V.}} + \dot{m}\,(a_{f_1} - a_{f_2}) - \dot{I}_{\text{C.V.}} = 0 \qquad (8.82)$$

where $a_{f_1} - a_{f_2} = (h_1 - h_2) - T_0(s_1 - s_2) + \frac{V_1^2 - V_2^2}{2} + g(Z_1 - Z_2)$ and $[1 - T_0/T_j]\,\dot{Q}_j$ = time rate of exergy transfer along with heat $\dot{Q}_j$ occurring at the location on the boundary where the instantaneous temperature is T_j.

For a single stream entering and leaving, the exergy balance gives

$$\underbrace{\left[1 - \frac{T_0}{T_\sigma}\right] \frac{\dot{Q}}{\dot{m}} + a_{f_1}}_{\text{Exergy in}} \; \underbrace{- \frac{\dot{W}}{\dot{m}} - a_{f_2}}_{\text{Exergy out}} = \underbrace{\frac{\dot{I}}{\dot{m}}}_{\text{Exergy loss}} \qquad (8.83)$$

8.12 SECOND LAW EFFICIENCY

A common measure on energy use efficiency is the first law efficiency, η_I. The first law efficiency is defined as the ratio of the output energy of a device to the input energy of the device. The first law is concerned only with the quantities of energy, and disregards the forms in which the energy exists. It does not also discriminate between the energies available at different temperatures. It is the second law of thermodynamics which provides a means of assigning a quality index to energy. The concept of available energy or exergy provides a useful measure of energy quality (Sec. 8.3).

With this concept it is possible to analyze means of minimizing the consumption of available energy to perform a given process, thereby ensuring the most efficient possible conversion of energy for the required task.

The second law efficiency, η_{II}, of a process is defined as the ratio of the minimum available energy (or exergy) which must be consumed to do a task divided by the actual amount of available energy (or exergy) consumed in performing the task.

$$\eta_{II} = \frac{\text{minimum exergy intake to perform the given task}}{\text{actual exergy intake to perform the same task}}$$

or
$$\eta_{II} = \frac{A_{\min}}{A} \tag{8.84}$$

where A is the availability or exergy.

A power plant converts a fraction of available energy A or $W_{\max}$ to useful work W. For the desired output of W, $A_{\min} = W$ and $A = W_{\max}$. Here,

$$I = W_{\max} - W \quad \text{and} \quad \eta_{II} = \frac{W}{W_{\max}} \tag{8.85}$$

Now
$$\eta_I = \frac{W}{Q_1} = \frac{W}{W_{\max}} \cdot \frac{W_{\max}}{Q_1} = \eta_{II} \cdot \eta_{\text{Carnot}} \tag{8.86}$$

$\therefore$
$$\eta_{II} = \frac{\eta_I}{\eta_{\text{Carnot}}} \tag{8.87}$$

Since $W_{\max} = Q_1\left(1 - \frac{T_0}{T}\right)$, Eq. (8.87) can also be obtained directly as follows

$$\eta_{II} = \frac{W}{Q_1\left(1 - \frac{T_0}{T}\right)} = \frac{\eta_I}{\eta_{\text{Carnot}}}$$

If work is involved, $A_{\min} = W$ (desired) and if heat is involved, $A_{\min} = Q\left(1 - \frac{T_0}{T}\right)$.

If solar energy Q_r is available at a reservoir storage temperature T_r and if quantity of heat Q_a is transferred by the solar collector at temperature T_a, then

$$\eta_I = \frac{Q_a}{Q_r}$$

and
$$\eta_{II} = \frac{\text{exergy output}}{\text{exergy input}} = \frac{Q_a\left(1 - \frac{T_0}{T_a}\right)}{Q_r\left(1 - \frac{T_0}{T_r}\right)} = \eta_I \frac{1 - \frac{T_0}{T_a}}{1 - \frac{T_0}{T_r}} \tag{8.88}$$

Table 8.1 shows availabilities, and both η_I and η_{II} expressions for several common thermal tasks.

Table 8.1

$T_p > T_a > T_0 > T_c$

Task	***Energy input***	
	Input shaft work, W_i	***Q from reservoir at T***
Produce work, W_0	$A = W_i$	$A = Q_r\left(1 - \frac{T_0}{T_r}\right)$
	$A_{\min} = W_0$	$A_{\min} = W_0$
	$\eta_I = \frac{W_0}{W_i}$	$\eta_I = \frac{W_0}{Q_r}$

Table 8.1 (*Continued*)

Task	*Energy input*	
	Input shaft work, W_i	*Q_r from reservoir at T_r*
	$\eta_{II} = \dfrac{A_{min}}{A}$	$\eta_{II} = \eta_I \cdot \dfrac{1}{1 - \dfrac{T_0}{T_r}}$
	$\eta_{II} = \eta_I$ (electric motor)	(heat engine)
Add heat Q_a to reservoir at T_a	$A = W_i$	$A = Q_r \left(1 - \dfrac{T_0}{T_r}\right)$
	$A_{min} = Q_a \left(1 - \dfrac{T_0}{T_a}\right)$	$A_{min} = Q_a \left(1 - \dfrac{T_0}{T_a}\right)$
	$^*\eta_I = \dfrac{Q_a}{W_i}$	$\eta_I = \dfrac{Q_a}{Q_r}$
	$^*\eta_{II} = \eta_I \left(1 - \dfrac{T_0}{T_a}\right)$	$\eta_{II} = \eta_I \dfrac{1 - \dfrac{T_0}{T_a}}{1 - \dfrac{T_0}{T_r}}$
	(heat pump)	(solar water heater)
Extract heat Q_c from cold reservoir at T_c (below ambient)	$A = W_i$	$A = Q_r \left(1 - \dfrac{T_0}{T_r}\right)$
	$A_{min} = Q_c \left(\dfrac{T_0}{T_c} - 1\right)$	$A_{min} = Q_c \left(\dfrac{T_0}{T_c} - 1\right)$
	$^*\eta_I = \dfrac{Q_c}{W_i}$	$^*\eta_I = \dfrac{Q_c}{Q_r}$
	$^*\eta_{II} = \eta_I \left(\dfrac{T_0}{T_c} - 1\right)$	$^*\eta_{II} = \eta_I \left(\dfrac{\dfrac{T_0}{T_c} - 1}{1 - \dfrac{T_0}{T_r}}\right)$
	(Refrigerator-electric motor driven)	(Refrigerator-heat operated)

* Strictly speaking, it is COP.

In the case of a heat pump, the task is to add heat Q_a to a reservoir to be maintained at temperature T_a and the input shaft work is W_i.

$$\text{COP} = \frac{Q_a}{W_i} = \eta_I, \text{ say } (\text{COP})_{\max} = \frac{T_a}{T_a - T_0} = \frac{Q_a}{W_i} = \frac{Q_a}{A_{\min}}$$

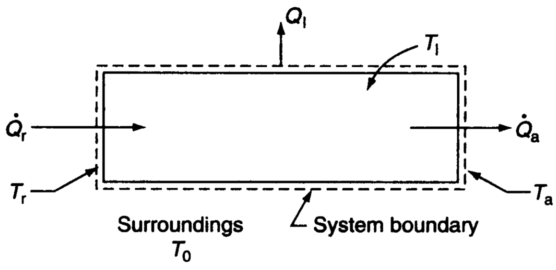
$$\therefore \quad A_{\min} = Q_a\left(1 - \frac{T_0}{T_a}\right)$$

$$\therefore \quad \eta_{II} = \frac{A_{\min}}{A} = \frac{Q_a\left(1 - \frac{T_0}{T_a}\right)}{W_i} \qquad \eta_{II} = \eta_I\left(1 - \frac{T_0}{T_a}\right) \tag{8.89}$$

Similarly, expressions of η_I and η_{II} can be obtained for other thermal tasks.

8.12.1 Matching End Use to Source

Combustion of a fuel releases the necessary energy for the tasks, such as space heating, process steam generation and heating in industrial furnaces. When the products of combustion are at a temperature much greater than that required by a given task, the end use is not well matched to the source and results in inefficient utilization of the fuel burned. To illustrate this, let us consider a closed system receiving a heat transfer $\dot{Q}_r$ at a source temperature T_r and delivering $\dot{Q}_a$ at a use temperature T_a (Fig. 8.19). Energy is lost to the surroundings by heat transfer at a rate $\dot{Q}_l$ across a portion of the surface at T_l. At a steady state the energy and availability rate balances become.



Efficient energy utilization from second law viewpoint

$$\dot{Q}_r = \dot{Q}_a + \dot{Q}_l \tag{8.90}$$

$$\dot{Q}_r\left(1 - \frac{T_0}{T_r}\right) = \dot{Q}_a\left(1 - \frac{T_0}{T_a}\right) + \dot{Q}_l\left(1 - \frac{T_0}{T_l}\right) + \dot{I} \tag{8.91}$$

Equation (8.90) indicates that the energy carried in by heat transfer $\dot{Q}_r$ is either used, $\dot{Q}_a$, or lost to the surroundings, $\dot{Q}_l$. Then

$$\eta_I = \frac{\dot{Q}_a}{\dot{Q}_r} \tag{8.92}$$

The value of η_I can be increased by increasing insulation to reduce the loss. The limiting value, when $\dot{Q}_l = 0$, is $\eta_I = 1$ (100%).

Equation (8.91) shows that the availability carried into the system accompanying the heat transfer $\dot{Q}_r$ is either transferred from the system accompanying the heat transfer $\dot{Q}_a$ and $\dot{Q}_l$ or destroyed by irreversibilities within the system, $\dot{I}$. Therefore,

$$\eta_{II} = \frac{\dot{Q}_a\left(1 - \frac{T_0}{T_a}\right)}{\dot{Q}_r\left(1 - \frac{T_0}{T_r}\right)} = \eta_I \frac{1 - \frac{T_0}{T_a}}{1 - \frac{T_0}{T_r}} \tag{8.93}$$

Both η_I and η_II indicate how effectively the input is converted into the product. The parameter η_I does this on energy basis, whereas η_II does it on an availability or exergy basis.

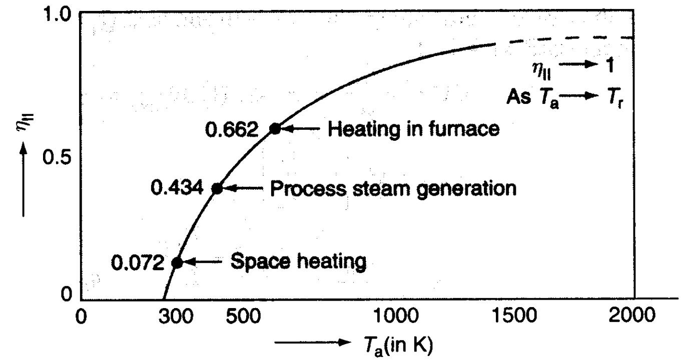


Fig. 8.20 *Effect of use temperature T_a on the second law efficiency ($T_r = 2200$ K, $T_0 = 300$ K, $n_l = 100\%$)*

For proper utilization of exergy, it is desirable to make η_I as close to unity as practical and also a good match between the source and use temperatures, T_r and T_a. Figure 8.20 demonstrates the second law efficiency against the use temperature T_a for an assumed source temperature $T_\text{r} = 2200$ K. It shows that η_II tends to unity (100%) as T_a approaches T_r. The lower the T_a, the lower becomes the value of η_II. Efficiencies for three applications, viz., space heating at $T_\text{a} = 320$ K, process steam generation at $T_\text{a} = 480$ K, and heating in industrial furnaces at $T_\text{a} = 700$ K, are indicated on the Figure. It suggests that fuel is used far more effectively in the high temperature use. An excessive temperatures gap between T_r and T_a causes a low η_II and an inefficient energy utilization. A fuel or any energy source is consumed efficiently when the first user temperature approaches the fuel temperature. This means that the fuel should first be used for high temperature applications. The heat rejected from these applications can then be cascaded to applications at lower temperatures, eventually to the task of, say, keeping a building warm. This is called *energy cascading* and ensures more efficient energy utilization.

8.12.2 Further Illustrations of Second Law Efficiencies

Second law efficiency of different components can be expressed in different forms. It is derived by using the exergy balance rate given below:

(*a*) *Turbines* The steady state exergy balance (Fig. 8.21) gives:

$$\frac{\dot{Q}}{\dot{m}}\left[1-\frac{T_0}{T_\sigma}\right]+a_{\text{f}_1} = \frac{\dot{W}}{\dot{m}}+a_{\text{f}_2}+\frac{\dot{I}}{\dot{m}}$$

If there is not heat loss,

$$a_{\text{f}_1}-a_{\text{f}_2} = \frac{\dot{W}}{\dot{m}}+\frac{\dot{I}}{\dot{m}} \tag{8.94}$$

The second law efficiency,
$$\eta_\text{II} = \frac{\dot{W}/\dot{m}}{a_{\text{f}_1}-a_{\text{f}_2}} \tag{8.95}$$

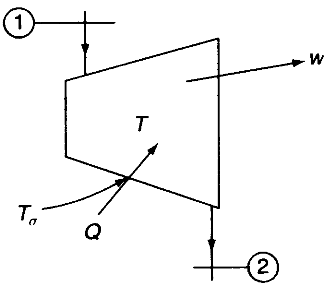


Fig. 8.21 *Exergy balance of a turbine*

(*b*) *Compressor and Pump* Similarly, for a compressor or a pump,

$$\frac{\dot{W}}{\dot{m}} = a_{\text{f}_2}-a_{\text{f}_1}+\frac{\dot{I}}{\dot{m}}$$

and
$$\eta_\text{II} = \frac{a_{\text{f}_2}-a_{\text{f}_1}}{-\dot{W}/\dot{m}} \tag{8.96}$$

(*c*) *Heat Exchanger* Writing the exergy balance for the heat exchanger, (Fig. 8.22)

$$\sum \left[1 - \frac{T_0}{T_j}\right] \dot{Q}_j - \dot{W}_{C.V.} + [\dot{m}_h\, a_{f_1} + \dot{m}_c\, a_{f_3}] - [\dot{m}_h\, a_{f_2} + \dot{m}_c\, a_{f_4}] - \dot{I}_{C.V.} = 0$$

If there is no heat transfer and work transfer,

$$\dot{m}_h\, [a_{f_1} - a_{f_2}] = \dot{m}_c\, [a_{f_4} - a_{f_3}] + \dot{I} \qquad (8.97)$$

$$\eta_{II} = \frac{\dot{m}_c\, [a_{f_4} - a_{f_3}]}{\dot{m}_h\, [a_{f_1} - a_{f_2}]} \qquad (8.98)$$

(*d*) *Mixing of Two Fluids* Exergy balance for the mixer (Fig. 8.23) gives:

$$\left[1 - \frac{T_0}{T_\sigma}\right] \dot{Q} + \dot{m}_1\, a_{f_1} + \dot{m}_2\, a_{f_2} = \dot{W}_{C.V.} + \dot{m}_3\, a_{f_3} + I_{C.V.}$$

If the mixing is adiabatic and since $\dot{W}_{C.V.} = 0$ and $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$.

$$\dot{m}_1\, [a_{f_1} - a_{f_3}] = \dot{m}_2\, [a_{f_3} - a_{f_2}] + \dot{I} \qquad (8.99)$$

and

$$\eta_{II} = \frac{\dot{m}_2\, [a_{f_3} - a_{f_2}]}{\dot{m}_1\, [a_{f_1} - a_{f_3}]} \qquad (8.100)$$

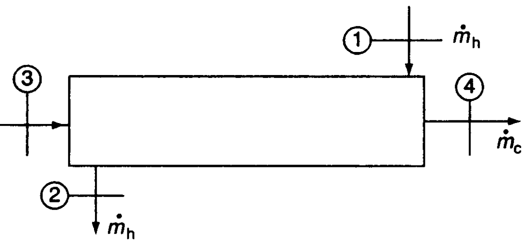


Fig. 8.22 *Exergy balance of a heat exchanger*

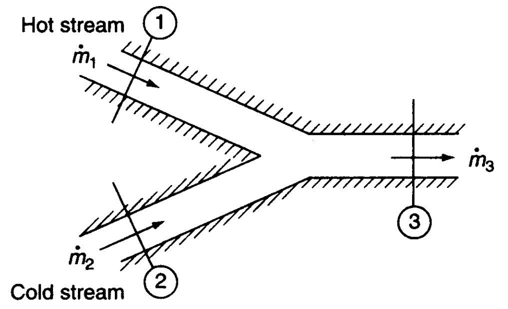


Fig. 8.23 *Exergy loss due to mixing*

8.13 COMMENTS ON EXERGY

The energy of the universe, like its mass, is constant. Yet at times, we are bombarded with speeches and articles on how to "conserve" energy. As engineers, we know that energy is always conserved. What is not conserved is the exergy, i.e., the useful work potential of the energy. Once the exergy is wasted, it can never be recovered. When we use energy (electricity) to heat our homes, we are not destroying any energy, we are merely converting it to a less useful form, a form of less exergy value.

The maximum useful work potential of a system at the specified state is called exergy which is a composite property depending on the state of the system and the surroundings. A system which is in equilibrium with its surroundings is said to be at the dead state having zero exergy.

The mechanical forms of energy such as KE and PE are entirely available energy or exergy. The exergy (W) of thermal energy (Q) of reservoirs (TER) is equivalent to the work output of a Carnot heat engine operating between the reservoir at temperature T and environment at T_0, i.e., $W = Q\left[1 - \frac{T_0}{T}\right]$.

The actual work W during a process can be determined from the first law. If the volume of the system changes during a process, part of this work (W_{surr}) is used to push the surrounding medium at constant pressure p_0 and it cannot be used for any useful purpose. The difference between the actual work and the surrounding work is called *useful work*, W_u

$$W_u = W - W_{surr} = W - p_0(V_2 - V_1)$$

W_{surr} is zero for cyclic devices, for steady flow devices, and for system with fixed boundaries (rigid walls).

The maximum amount of useful work that can be obtained from a system as it undergoes a process between two specified states is called *reversible work*, W_{rev}. If the final state of the system is the dead state, the reversible work and the exergy become identical.

The difference between the reversible work and useful work for a process is called *irreversibility*.

$$I = W_{rev} - W_u = T_0 S_{gen} \qquad \dot{I} = T_0 \dot{S}_{gen}$$

For a total reversible process, $W_{rev} = W_u$ and $I = 0$.

The first law efficiency alone is not a realistic measure of performance for engineering devices. Consider two heat engines, having e.g., a thermal efficiency of, say, 30%. One of the engines (A) is supplied with heat Q from a source at 600 K and the other engine (B) is supplied with the same amount of heat Q from a source at 1000 K. Both the engines reject heat to the surroundings at 300 K.

$$(W_A)_{rev} = Q\left(1 - \frac{300}{600}\right) = 0.5Q, \text{ while } (W_A)_{act} = 0.3Q$$

Similarly,

$$(W_B)_{rev} = Q\left(1 - \frac{300}{1000}\right) = 0.7Q, \text{ and } (W_B)_{act} = 0.3Q$$

At first glance, both engines seem to convert the same fraction of heat, that they receive, to work, thus performing equally well from the viewpoint of the first law. However, in the light of second law, the engine B has a greater work potential (0.7 Q) available to it and thus should do a lot better than engine A. Therefore, it can be said that engine B is performing poorly relative to engine A, even though both have the same thermal efficiency.

To overcome the deficiency of the first law efficiency, a second law efficiency η_{II} can be defined as the ratio of actual thermal efficiency to the maximum possible thermal efficiency under the same conditions:

$$\eta_{II} = \frac{\eta_I}{\eta_{rev}}$$

So, for engine A, $\eta_{II} = 0.3/0.5 = 0.60$
and for engine B, $\eta_{II} = 0.3/0.7 = 0.43$

Therefore, the engine A is converting 60% of the available work potential (exergy) to useful work. This is only 43% for the engine B. Therefore,

$$\eta_{II} = \frac{\eta_{act}}{\eta_{rev}} = \frac{W_u}{W_{rev}} \text{ (for heat engines and other work producing devices)}$$

$$\eta_{II} = \frac{COP}{COP_{rev}} = \frac{W_{rev}}{W_u} \text{ (for refrigerators, heat pumps and other work absorbing devices)}$$

The exergies of a closed system (ϕ) and a flowing fluid system (ψ) are given on unit mass basis:

$$\phi = (u - u_0) - T_0(s - s_0) + p_0(v - v_0) \text{ kJ/kg}$$

$$\Psi = (h - h_0) - T_0(s - s_0) + \frac{V^2}{2} + gz \text{ kJ/kg}$$

Reversible Work Expressions

(*a*) ***Cyclic Devices***

$$W_{rev} = \eta_{rev}\, Q_1 \text{ (Heat engines)}$$

$$-W_{\text{rev}} = \frac{Q_2}{(COP_{\text{rev}})_{\text{Ref.}}} \text{ (Refrigerators)}$$

$$-W_{\text{rev}} = \frac{Q_2}{(COP_{\text{rev}})_{\text{HP}}} \text{ (Heat pumps)}$$

(*b*) ***Closed System*** $W_{\text{rev}} = U_1 - U_2 - T_0(S_1 - S_2) + p_0(V_1 - V_2) = m(\phi_1 - \phi_2)$

(*c*) ***Steady Flow System*** (***single stream***)

$$W_{\text{rev}} = m\left[\left(h_1 + \frac{V_1^2}{2} + gz_1 - T_0 s_1\right) - \left(h_2 + \frac{V_2^2}{2} + gz_2 - T_0 s_2\right)\right] = m(\Psi_1 - \Psi_2)$$

When the system exchanges heat with another reservoir at temperature T_k other than the atmosphere,

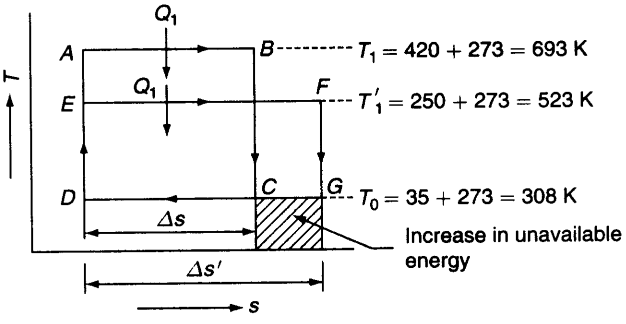
$$\dot{W}_{\text{rev}} = \dot{m}(\Psi_1 - \Psi_2) + \dot{Q}_k\left(1 - \frac{T_0}{T_k}\right)$$

The first law efficiency is defined as the ratio of energy output and energy input, while their difference is the energy loss. Likewise, the second law efficiency is defined as the ratio of exergy output and exergy input and their difference is irreversibility. By reducing energy loss, first law efficiency can be improved. Similarly, by reducing irreversibilities, the second law efficiency can be enhanced.

Solved Examples

Example 8.1

In a certain process, a vapour, while condensing at 420°C, *transfers heat to water evaporating at* 250°C. *The resulting steam is used in a power cycle which rejects heat at* 35°C. *What is the fraction of the available energy in the heat transferred from the process vapour at* 420°C *that is lost due to the irreversible heat transfer at* 250°C?



[illegible]

Solution *ABCD* (Fig.Ex. 8.1) would have been the power cycle, if there was no temperature difference between the vapour condensing and the water evaporating and the area under *CD* would have been the unavailable energy. *EFGD* is the power cycle when the vapour condenses at 420°C and the water evaporates at 250°C. The unavailable energy becomes the area under *DG*. Therefore, the increase in unavailable energy due to irreversible heat transfer is represented by the area under *CG*.

Now $Q_1 = T_1\, \Delta S = T'_1\, \Delta S'$

$$\frac{\Delta S'}{\Delta S} = \frac{T_1}{T_1'}$$

$$W' = \text{work done in cycle } ABCD = (T_1 - T_0)\,\Delta S$$

$$W' = \text{Work done in cycle } EFGD = (T'_1 - T_0)\,\Delta S'$$

The fraction of energy that becomes unavailable due to irreversible heat transfer

$$= \frac{W - W'}{W} = \frac{T_0\,(\Delta S' - \Delta S)}{(T_1 - T_0)\,\Delta S} = \frac{T_0\left(\dfrac{\Delta S'}{\Delta S} - 1\right)}{(T_1 - T_0)}$$

$$= \frac{T_0\,(T_1 - T_1')}{T_1'\,(T_1 - T_0)} = \frac{308\,(693 - 523)}{523\,(693 - 308)}$$

$$= 0.26$$

Ans.

Example 8.2

In a steam boiler, hot gases from a fire transfer heat to water which vaporizes at constant temperature. In a certain case, the gases are cooled from 1100°C *to* 550°C *while the water evaporates at* 220°C. *The specific heat of gases is* 1.005 *kJ/kgK, and the latent heat of water at* 220°C, *is* 1858.5 *kJ/kg. All the heat transferred from the gases goes to the water. How much does the total entropy of the combined system of gas and water increase as a result of the irreversible heat transfer? Obtain the result on the basis of* 1 *kg of water evaporated.*

If the temperature of the surroundings is 30°C, *find the increase in unavailable energy due to irreversible heat transfer.*

Solution Gas ($\dot{m}_g$) is cooled from state 1 to state 2 (Fig. Ex. 8.2). For reversible heat transfer, the working fluid (w.f.) in the heat engine having the same c_p would have been heated along 2–1, so that at any instant, the temperature difference between gas and the working fluid is zero. Then 1–b would have been the expansion of the working fluid down to the lowest possible temperature T_0, and the amount of heat rejection would have been given by the area *abcd*.

When water evaporates at 220°C as the gas gets cooled from 1100°C to 550°C, the resulting power cycle has an unavailable energy represented by the area *aefd*. The increase in unavailable energy due to irreversible heat transfer is thus given by area *befc*.

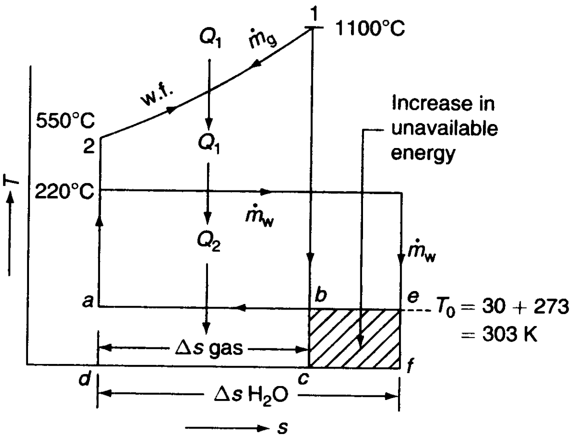


Fig. Ex. 8.2

Entropy increase of 1 kg water

$$(\Delta S)_{\text{water}} = \frac{\text{Latent heat absorbed}}{T} = \frac{1858.5}{(273+220)} = 3.77 \text{ kJ/kgK}$$

Q_1 = Heat transferred from the gas = Heat absorbed by water during evaporation

$$= m_g c_{p_g} (1100-550) = 1 \times 1858.5 \text{ kJ}$$

$$\therefore \qquad \dot{m}_g\, c_{p_g} = \frac{1858.5}{550} = 3.38 \text{ kJ/°C}$$

$$\Delta S_{\text{gas}} = \int_{T_{g1}}^{T_{g2}} \frac{\text{đ}Q}{T} = \int_{T_{g1}}^{T_{g2}} \dot{m}_g\, c_{p_g} \frac{\mathrm{d}T}{T}$$

$$= \dot{m}_g\, c_{p_g} \ln \frac{T_{g2}}{T_{g1}} = 3.38 \ln \frac{823}{1373}$$

$$= -3.38 \times 0.51 = -1.725 \text{ kJ/K}$$

$$\therefore \qquad \Delta S_{\text{total}} = (\Delta S)_{\text{water}} + (\Delta S)_{\text{gas}} = 3.77 - 1.725 = 2.045 \text{ kJ/K}$$ *Ans.*

Increase in unavailable energy $= T_0\,(\Delta S)_{\text{total}} = 303 \times 2.045 = 620$ kJ *Ans.*

Example 8.3

Calculate the available energy in 40 kg of water at 75°C with respect to the surroundings at 5°C, the pressure of water being 1 atm.

Solution If the water is cooled at a constant pressure of 1 atm from 75°C to 5°C (Fig. Ex. 8.3) the heat given up may be used as a source for a series of Carnot engines each using the surroundings as a sink. It is assumed that the amount of energy received by any engine is small relative to that in the source and the temperature of the source does not change while heat is being exchanged with the engine.

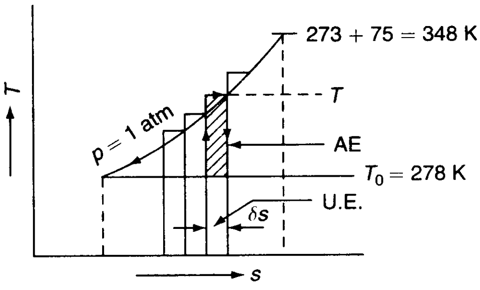


Fig. Ex. 8.3

Let us consider that the source has fallen to temperature T, at which level there operates a Carnot engine which takes in heat at this temperature and rejects heat at $T_0 = 278$ K. If δs is the entropy change of water, the work obtainable is

$$\delta W = -\,m\,(T - T_0)\,\delta s$$

where δs is negative.

$$\delta W = -\,40\,(T - T_0)\,\frac{c_p\,\delta T}{T}$$

$$\therefore \qquad = -\,40\,c_p \left(1 - \frac{T_0}{T}\right) \delta T$$

With a very great number of engines in the series, the total work (maximum) obtainable when the water is cooled from 348 K to 278 K would be

$$W_{(\max)} = \text{A.E.} = -\lim \sum_{348}^{278} 40 c_p \left(1 - \frac{T_0}{T}\right) \delta T$$

$$= \int_{278}^{348} 40\, c_p \left(1 - \frac{T_0}{T}\right) \mathrm{d}T = 40\, c_p \left[(348 - 278) - 278 \ln \frac{348}{278}\right] = 40 \times 4.2\,(70 - 62) = 1340 \text{ kJ} \quad \textit{Ans.}$$

$$Q_1 = 40 \times 4.2\,(348 - 278) = 11{,}760 \text{ kJ U.E.} = Q_1 - W_{(\max)} = 11{,}760 - 1340 = 10{,}420 \text{ kJ}$$

Example 8.4

Calculate the decrease in available energy when 25 kg of water at 95°C mix with 35 kg of water at 35°C, the pressure being taken as constant and the temperature of the surroundings being 15°C (c_p of water = 4.2 kJ/kg K).

Solution The available energy of a system of mass m, specific heat c_p, and at temperature T, is given by

$$\text{A.E.} = mc_p \int_{T_0}^{T} \left(1 - \frac{T_0}{T}\right) \mathrm{d}T$$

$\therefore$ $(\text{A.E.})_{25}$ = Available energy of 25 kg of water at 95°C $= 25 \times 4.2 \int_{273+15}^{273+95} \left(1 - \frac{288}{T}\right) \mathrm{d}T$

$$= 105 \left[(368 - 288) - 288 \ln \frac{368}{288}\right] = 987.49 \text{ kJ}$$

$(\text{A.E.})_{35}$ = Available energy of 35 kg of water at 35°C

$$= 147 \left[(308 - 288) - 288 \ln \frac{308}{288}\right] = 97.59 \text{ kJ}$$

Total available energy

$$(\text{A.E.})_{\text{total}} = (\text{A.E.})_{25} + (\text{A.E.})_{35} = 987.49 + 97.59 = 1085.08 \text{ kJ}$$

After mixing, if t is the final temperature

$$25 \times 4.2\,(95 - t) = 35 \times 4.2\,(t - 35)$$

$$\therefore \quad t = \frac{25 \times 95 \times 35 \times 35}{25 + 35} = 60°\text{C}$$

Total mass after mixing = 25 + 35 = 60 kg

$(\text{A.E.})_{60}$ = Available energy of 60 kg of water at 60°C

$$= 4.2 \times 60 \left[(333 - 288) - 288 \ln \frac{333}{288}\right] = 803.27 \text{ kJ}$$

$\therefore$ Decrease in available energy due to mixing

= Total available energy before mixing– Total available energy after mixing
= 1085.08 – 803.27 = 281.81 kJ *Ans.*

Example 8.5

The moment of inertia of a flywheel is 0.54 kg · m^2 and it rotates at a speed 3000 RPM in a large heat insulated system, the temperature of which is 15°C. If the kinetic energy of the flywheel is dissipated as frictional heat at the shaft bearings which have a water equivalent of 2 kg, find the rise in the temperature of the bearings when the flywheel has come to rest. Calculate the greatest possible amount of this heat which may be returned to the flywheel as high-grade energy, showing how much of the original kinetic energy is now unavailable. What would be the final RPM of the flywheel, if it is set in motion with this available energy?

Solution Initial angular velocity of the flywheel

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi \times 3000}{60} = 314.2 \text{ rad/s}$$

Initial available energy of the flywheel $= (\text{K.E.})_{\text{initial}} = \frac{1}{2} I\omega_1^2 = 0.54 \text{ kg m}^2 \times (314.2)^2 \frac{\text{rad}^2}{\text{s}^2}$

$$= 2.66 \times 10^4 \text{ Nm} = 26.6 \text{ kJ}$$

When this K.E. is dissipated as frictional heat, if Δt is the temperature rise of the bearings, we have water equivalent of the bearings × rise in temperature = 26.6 kJ

$$\therefore \quad \Delta t = \frac{26.6}{2 \times 4.187} = 3.19°\text{C} \qquad \textit{Ans.}$$

∴ Final temperature of the bearings

$$t_f = 15 + 3.19 = 18.19°\text{C}$$

The maximum amount of energy which may be returned to the flywheel as high-grade energy is

$$\text{A.E.} = 2 \times 4.187 \int_{288}^{291.19} \left(1 - \frac{288}{T}\right) dT$$

$$= 2 \times 4.187 \left[(291.19 - 288) - 288 \ln \frac{291.19}{288}\right] = 0.1459 \text{ kJ}$$

The amount of energy rendered unavailable is

$$\text{U.E.} = (\text{A.E.})_{\text{initial}} - (\text{A.E.})_{\text{returnable as high energy}} = 26.6 - 0.1459 = 26.4541 \text{ kJ}$$

Since the amount of energy returnable to the flywheel is 0.146 kJ, if ω_2 is the final angular velocity, and the flywheel is set in motion with this energy

$$0.146 \times 10^3 = \frac{1}{2} \times 0.54\, \omega_2^2$$

$$\therefore \quad \omega_2^2 = \frac{146}{0.27} = 540.8$$

$$\therefore \quad \omega_2 = 23.246 \text{ rad/s}$$

If N_2 is the final RPM of the flywheel

$$\omega_2 = 23.246 = \frac{2\pi N_2}{60}$$

or

$$N_2 = \frac{23.246 \times 60}{2 \times \pi} = 222 \text{ RPM} \qquad \textit{Ans.}$$

Example 8.6

Two kg of air at 500 kPa, 80°C expands adiabatically in a closed system until its volume is doubled and its temperature becomes equal to that of the surroundings which is at 100 kPa, 5°C. For this process, determine (a) the maximum work, (b) the change in availability, and (c) the irreversibility. For air, take c_v = 0.718 kJ/kg K, $u = c_v T$ where c_v is contant, and $pV = mRT$ where p is pressure in kPa, V volume in m^3, m mass in kg, R a constant equal to 0.287 kJ/kg K, and T temperature in K.

Solution From the property relation

$$TdS = dU + pdV$$

the entropy change of air between the initial and final states is

$$\int_1^2 dS = \int_1^2 \frac{mc_v dT}{T} + \int_1^2 \frac{mR dV}{V}$$

or $$S_2 - S_1 = mc_v \ln \frac{T_2}{T_1} + mR \ln \frac{V_2}{V_1}$$

From Eq. (8.32), $$W_{max} = (U_1 - U_2) - T_0 (S_1 - S_2)$$

$$= m \left[c_v (T_1 - T_2 + T_0 \left(c_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}\right)\right]$$

$$= 2 \left[0.718(80-5) + 278 \left(0.718 \ln \frac{278}{353} + 0.287 \ln \frac{2}{1}\right)\right]$$

$$= 2 [53.85 + 278 (- 0.172 + 0.199)]$$

$$= 2 (53.85 + 7.51) = 122.72 \text{ kJ}$$ *Ans.* (a)

From Eq. (8.42), the change in availability = $\phi_1 - \phi_2$

$$= (U_1 - U_2) - T_0(S_1 - S_2) + p_0(V_1 - V_2) = W_{max} + p_0(V_1 - V_2) = 122.72 + p_0(V_1 - 2V_1)$$

$$= 122.72 - 100 \times \frac{2 \times 0.287 \times 353}{500} = 82.2 \text{ kJ}$$ *Ans.* (b)

The irreversibility $I = W_{max,\ useful} - W_{act}$

From the first law, $W_{act} = Q - \Delta U = -\Delta U = U_1 - U_2$

$\therefore$ $$I = U_1 - U_2 - T_0(S_1 - S_2) - U_1 + U_2$$

$$= T_0(S_2 - S_1) = T_0(\Delta S)_{system}$$

For adiabatic process, $(\Delta S)_{surr} = 0$

$$I = T_0 \left[mc_v \ln \frac{T_2}{T_1} + mR \ln \frac{V_2}{V_1}\right]$$

$$= 278 \times 2 \left[0.718 \ln \frac{278}{353} + 0.287 \ln 2\right]$$

$$= 278 \times 2 (- 0.172 + 0.199) = 15.2 \text{ kJ}$$ *Ans.*(c)

Example 8.7

Air expands through a turbine from 500 kPa, 520°C to 100 kPa, 300°C. During expansion 10 kJ/kg of heat is lost to the surroundings which is at 98 kPa, 20°C. Neglecting the K.E. and P.E. changes, determine per kg of air (a) the decrease in availability, (b) the maximum work and (c) the irreversibility. For air, take $c_p = 1.005$ *kJ/kg K,* $h = c_p T$ *where* c_p *is constant, and the p, V and T relation as in Example 8.6.*

Solution From the property relation

$$TdS = dH - Vdp$$

the entropy change of air in the expansion process is

$$\int_1^2 dS = \int_1^2 \frac{mc_p dT}{T} - \int_1^2 \frac{mR dp}{p}$$

or $$S_2 - S_1 = mc_p \ln \frac{T_2}{T_1} - mR \ln \frac{p_2}{p_1}$$

For 1 kg of air, $$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

From Eq. (8.30), the change in availability

$$\Psi_1 - \Psi_2 = b_1 - b_2 = (h_1 - T_0 s_1) - (h_2 - T_0 s_2) = (h_1 - h_2) - T_0(s_1 - s_2)$$

$$= c_p (T_1 - T_2) - T_0 \left(R \ln \frac{p_2}{p_1} - c_p \ln \frac{T_2}{T_1} \right)$$

$$= 1.005\,(520 - 300) - 293 \left(0.287 \ln \frac{1}{5} - 1.005 \ln \frac{573}{793} \right)$$

$$= 1.005 \times 220 - 293\,(0.3267 - 0.4619) = 221.1 + 39.6$$

$$= 260.7 \text{ kJ/kg}$$ *Ans.* (a)

The maximum work is $W_{\max}$ = change in availability = $\Psi_1 - \Psi_2$

$= 260.7$ kJ/kg *Ans.* (b)

From S.F.E.E., $$Q + h_1 = W + h_2$$

$$W = (h_1 - h_2) + Q = c_p (T_1 - T_2) + Q$$

$$= 1.005\,(520 - 300) - 10 = 211.1 \text{ kJ/kg}$$

The irreversibility $$I = W_{\max} - W = 260.7 - 211.1 = 49.6 \text{ kJ/kg}$$ *Ans.* (c)

Alternatively, $$I = T_0\,(\Delta S_{\text{system}} + \Delta S_{\text{surr}})$$

$$= 293 \left(1.005 \ln \frac{573}{793} - 0.287 \ln \frac{1}{5} + \frac{10}{293} \right)$$

$$= 293 \times 0.1352 + 10 = 49.6 \text{ kJ/kg}$$ *Ans.* (d)

Example 8.8

An air preheater is used to cool the products of combustion from a furnace while heating the air to be used for combustion. The rate of flow of products is 12.5 *kg/s and the products are cooled from* 300 *to* 200°C, *and for the products at this temperature* $c_p = 1.09$ *kJ/kg K. The rate of air flow is* 11.5 *kg/s, the initial air temperature is* 40°C, *and for the air* $c_p = 1.005$ *kJ/kg K.* (*a*) *Estimate the initial and final availability of the products.* (*b*) *What is the irreversibility for the process?* (*c*) *If the heat transfer from the products occurs reversibly through heat engines, what is the final temperature of the air? What is the power developed by the heat engine? Take* $T_0 = 300$ *K and neglect pressure drop for both the fluids and heat transfer to the surroundings.*

Solution (a) Ψ_1 = initial availability of the products

$$= (h_1 - h_0) - T_0\,(S_1 - S_0) = c_{p_g}(T_{g_1} - T_0) - T_0 c_{p_g} \ln \frac{T_{g_1}}{T_0}$$

$$= 1.09\,(573 - 300) - 300 \times 1.09 \ln \frac{573}{300} = 297.57 - 211.6 = 39.68 \text{ kJ/kg}$$

Ψ_2 = final availability of the products

$$= (h_2 - h_0) - T_0(s_2 - s_0) = 1.09\,(473 - 300) - 300 \times 1.09 \times \ln \frac{473}{300}$$

$$= 188.57 - 148.89 = 39.68 \text{ kJ/kg}$$

(b) Decrease in availability of the products $= \Psi_1 - \Psi_2 = (h_1 - h_2) - T_0(s_1 - s_2)$

$$= 1.09\,(573 - 473) - 300 \times 1.09 \ln \frac{573}{473}$$

$$= 109 - 62.72 = 46.28 \text{ kJ/kg}$$

By making an energy balance for the air preheater [Fig. Ex. 8.8(a)].

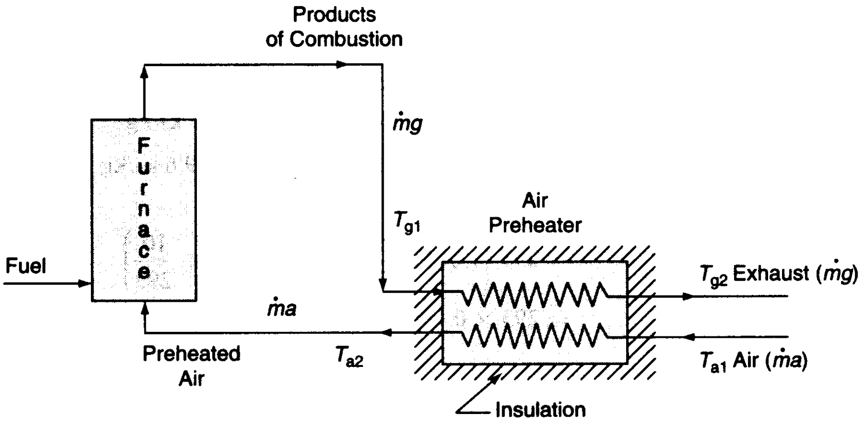
$$\dot{m}_g\, c_{p_g}(T_{g_1} - T_{g_2}) = \dot{m}_a\, c_{p_a}(T_{a_2} - T_{a_1})$$

$$12.5 \times 1.09\,(573 - 473) = 11.15 \times 1.005(T_{a_2} - 313)$$

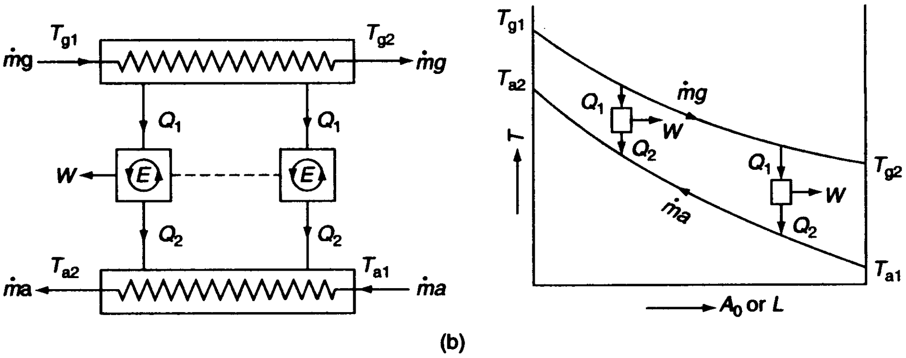
$$\therefore \qquad T_{a_2} = \frac{12.5 \times 109}{11.5 \times 1.005} + 313 = 430.89 \text{ K}$$

Increase in availability for air $= \Psi_2 - \Psi_1 = (h_2 - h_1) - T_0(s_2 - s_1)$

$$= c_{p_a}(T_{a_2} - T_{a_1}) - T_0 c_{p_a} \ln \frac{T_{a_2}}{T_{a_1}}$$



(a)



(b)

$$= 1.005 \times (430.89 - 313) - 300 \times 1.005 \ln \frac{430.89}{313}$$

$$= 118.48 - 96.37 = 22.11 \text{ kJ/kg}$$

$\therefore$ Irreversibility of the process $= 12.5 \times 46.28 - 11.5 \times 22.11$

$$= 578.50 - 254.27 = 324.23 \text{ kW}$$

(c) Let us assume that heat transfer from the products to air occurred through heat engines reversibly as shown in Fig. Ex. 8.8(b).

For reversible heat transfer,

$$\Delta \dot{S}_{\text{univ}} = 0$$

$$\Delta \dot{S}_{\text{sys}} + \Delta \dot{S}_{\text{surr}} = 0$$

$$\Delta \dot{S}_{\text{gas}} + \Delta \dot{S}_{\text{air}} = 0$$

$$\Delta \dot{S}_{\text{gas}} = -\Delta \dot{S}_{\text{air}}$$

$$\dot{m}_g c_{p_g} \ln \frac{T_{g2}}{T_{g1}} = -\dot{m}_a c_{p_a} \ln \frac{T_{a2}}{T_{a1}}$$

$$12.5 \times 1.09 \ln \frac{473}{573} = -11.5 \times 1.005 \ln \frac{T_{a2}}{313}$$

$$\therefore \quad T_{a2} = 392.41 \text{ K}$$

Rate of heat supply from the gas to the working fluid in the heat engine,

$$\dot{Q}_1 = \dot{m}_g c_{pg} (T_{g1} - T_{g2}) = 12.5 \times 1.09 (573 - 473) = 1362.50 \text{ kW}$$

Rate of heat rejection from the working fluid in the heat engine to the air,

$$\dot{Q}_2 = \dot{m}_a c_{p_a} (T_{a2} - T_{a1}) = 11.5 \times 1.005 (392.41 - 313) = 917.78 \text{ kW}$$

Total power developed by the heat engine

$$\dot{W} = \dot{Q}_1 - \dot{Q}_2 = 1362.50 - 917.78 = 444.72 \text{ kW}$$

Example 8.9

A gas is flowing through a pipe at the rate of 2 *kg/s. Because of inadequate insulation the gas temperature decreases from* 800 *to* 790°C *between two sections in the pipe. Neglecting pressure losses, calculate the irreversibility rate (or rate of energy degradation) due to this heat loss. Take* $T_0 = 300$ *K and a constant* $c_p = 1.1$ *kJ/kg K.*

For the same temperature drop of 10°C *when the gas cools from* 80°C *to* 70°C *due to heat loss, what is the rate of energy degradation? Take the same values of* T_0 *and* c_p*. What is the inference you can draw from this example?*

Solution

$$\dot{S}_{\text{gen}} = \dot{S}_{\text{sys}} - \frac{\dot{Q}}{T_0}$$

$$= \dot{m}(s_2 - s_1) - \frac{\dot{m}c_p(T_2 - T_1)}{T_0}$$

Irreversibility rate = rate of energy degradation = rate of exergy loss

$$\dot{I} = T_0\dot{S}_{gen} = \dot{m}T_0(s_2 - s_1) - \dot{m}c_p(T_2 - T_1)$$

$$= \dot{m}c_p\left[(T_1 - T_2) - T_0 \ln\frac{T_1}{T_2}\right]$$

$$= 2 \times 1.1\left[(1073 - 1063) - 300 \ln\frac{1073}{1063}\right] = 15.818 \text{ kW}$$

When the same heat loss occurs at lower temperature

$$\dot{I} = 2 \times 1.1\left[(353 - 343) - 300 \ln\frac{353}{343}\right] = 3.036 \text{ kW}$$

It is thus seen that irreversibility rate or exergy destruction is more when the same heat loss occurs at higher temperature. Irreversibility rate decreases as the temperature of the gas decreases. Quantitatively, the heat loss may be the same, but qualitatively, it is different.

Example 8.10

An ideal gas is flowing through an insulated pipe at the rate of 3 *kg/s. There is a* 10% *pressure drop from inlet to exit of the pipe. What is the rate of exergy loss because of the pressure drop due to friction? Take* $R = 0.287$ *kJ/kg K and* $T_0 = 300$ *K.*

Solution Rate of entropy generation from Eq. (8.68),

$$\dot{S}_{gen} = \dot{m}R\frac{\Delta p}{p_1} = 3 \times 0.287 \frac{0.10 p_1}{p_1} = 0.0861 \text{ kW/K}$$

Rate of exergy loss $\dot{I} = T_0 \dot{S}_{gen} = 300 \times 0.0861 = 25.83$ kW

Example 8.11

Water at 90°C *flowing at the rate of* 2 *kg/s mixes adiabatically with another stream of water at* 30°C *flowing at the rate of* 1 *kg/s. Estimate the entropy generation rate and the rate of exergy loss due to mixing. Take* $T_0 = 300$ *K.*

Solution

$$\dot{m} = \dot{m}_1 + \dot{m}_2 = 2 + 1 = 3 \text{ kg/s}$$

Here

$$x = \frac{\dot{m}_1}{\dot{m}} = \frac{2}{3} = 0.67$$

$$\tau = \frac{T_2}{T_1} = \frac{303}{363} = 0.835$$

From Eq. (8.76),

$$\dot{S}_{gen} = \dot{m}c_p \ln\frac{x + \tau(1 - x)}{\tau^{1-x}}$$